

**A family of sequences generated by
eliminating some prime factors from
the sequence of the naturals**

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ABSTRACT

This paper starts with an interesting property of the sequence of the naturals $\{A_n\}$, i.e. $\{A_n\} := \{1, 2, 3, \dots\}$. Considering the generated sequence $\{L_n\}$ by eliminating all the factor 2 in A_n if it has, we show that the partial sum of the new sequence $\{L_n\}$ from 2^{n-1} th term to $(2^n - 1)$ th term is square number 2^{2n-2} .

This property is further extended to the more general case: $\{H(n)\}$ is the sequence with each term $H(n)$ generated according to the following transforming rule:

ER ($\{p_1, p_2, \dots, p_m\}$): Given m primes $p_i, i = 1, 2, \dots, m$, for each $i \in \{1, 2, \dots, m\}$, if A_n has factor p_i , then substitute A_n with A_n / p_i until it does not have factor p_i .

We focus on the partial sum of $\{H(n)\}$. In **section one**, we research on Question 1: Given one prime p , we eliminate all the factor p in $\{A_n\}$ according to the transforming rule **ER** ($\{p\}$). Then the new sequence $\{L_n\}$ obtained has the property as follows:

$$\sum_{i=kp^{n-1}}^{kp^n-1} L_i = \frac{(p-1)}{2} k^2 p^{2n-1}.$$

The main results in this section are Theorem 1, 2 and the relative corollaries.

In **section two**, we mainly study Question 2: Given two prime p, q , we eliminate all factors p and q in $\{A_n\}$ according to the transforming rule **ER** ($\{p, q\}$). Then the new sequence $\{H(n)\}$ obtained has the property as follows:

$$\sum_{i=1}^{kp^r q^s} H(i) = \frac{k^2 pq(p^{2r} - 1)(q^{2s} - 1)}{2(p+1)(q+1)} + \sum_{i=1}^{kp^r} H(i) + \sum_{i=1}^{kq^s} H(i) - \sum_{i=1}^k H(i)$$

$$\text{(or } \sum_{i=kp^r}^{kp^r q^s} H(i) = \frac{k^2 pq(p^{2r} - 1)(q^{2s} - 1)}{2(p+1)(q+1)} + \sum_{i=k}^{kq^s} H(i) \text{)}.$$

We also focus on the number of valued “1” point of $\{H(n)\}$ from this section, where the term in $\{A_n\}$ which becomes the number 1 in $\{H(n)\}$ is called a valued “1” point of

$\{H(n)\}$. Let $Y(n) = \{i \mid H(i) = 1, i \in N, i \leq n\}$, we have

$$|Y(kp^r q^s)| = |Y(kp^r)| + |Y(kq^s)| - |Y(k)| + rs,$$

The main results in this section are Theorem 3, 4, 5, 6 and the relative corollaries.

In **section three**, we extend to the case of given m primes $p_i, i = 1, 2, \dots, m$, i.e. Question 3, and obtain similar results in Theorem 7, 8, 9, 10 and the relative corollaries.

In **section four**, we propose Question 4 and obtain several concise inequalities to estimate the partial sum of the first n terms and the number of valued "1" point in the first n terms. Furthermore, we derive a new proof for the noted fact that the set of primes is infinite.

For given two primes case, we have (Theorem 11, 12)

$$\frac{pq}{2(p+1)(q+1)} x^2 - 2x < \sum_{i \leq x} H(i) < \frac{pq}{2(p+1)(q+1)} x^2 + 2x$$

$$\frac{1}{2} \frac{\log x}{\log p} \left(\frac{\log x}{\log q} + 1 \right) < |Y(x)| < \frac{1}{2} \left(\frac{\log x}{\log p} + 1 \right) \left(\frac{\log x}{\log q} + 2 \right) \quad (\text{where } p < q)$$

For given m primes case, we have (Theorem 13, 14)

$$\frac{1}{2} \prod_{i=1}^m \frac{p_i}{p_i + 1} x^2 - 2^{m-1} x < \sum_{i \leq x} H(i) < \frac{1}{2} \prod_{i=1}^m \frac{p_i}{p_i + 1} x^2 + 2^{m-1} x.$$

$$|Y(x)| = \frac{1}{m!} \frac{\log^m x}{\prod_{i=1}^m \log p_i} + O(\log^{m-1} x).$$

Finally, we point out that the multiplicative functions $H(n), H'(n)$ have further research value.

Key words: sequence of the naturals, multiplicative function, partial sum, valued "1" point.

Section 1

The set of naturals is buried with numerous secrets. Among those, I find an interesting rule. Let $\{A_n\}$ be the sequence of the naturals i.e. $\{A_n\} = \{1, 2, 3, \dots, n, \dots\}$. We obtain a new sequence $\{L_n\}$ by eliminating all the factor 2 for every term of $\{A_n\}$ according to the following rule:

- (1) If A_n has factor 2, then we substitute A_n with $A_n/2$ until it does not have factor 2;
- (2) If A_n does not have factor 2, then keep it unchanged.

Thus, we obtain a new sequence $\{L_n\}$ as follows:

$$1, 1, 3, 1, 5, 3, 7, 1, 9, 5, 11, 3, 13, 7, 15, 1, 17, 9, 19, 5, 21, 11, 23, 3, 25, 13, 27, 7, 29, 15, 31, 1, 33, 17, 35, 9, 37, 19, 39, 5, 41, \dots \quad (1)$$

We can dig out some secrets if we divide it into groups with 1 being the first term of each group as follows:

$$(1), (1, 3), (1, 5, 3, 7), (1, 9, 5, 11, 3, 13, 7, 15), (1, 17, 9, 19, 5, 21, 11, 23, 3, 25, 13, 27, 7, 29, 15, 31), (1, 33, 17, 35, 9, 37, 19, 39, 5, 41, \dots)$$

Summing up each group, we derive that the sum of these groups are 1, 4, 16, 64, ... respectively. Thus we have the following conjecture:

Conjecture 1. The sum of the new sequence $\{L_n\}$ from 2^{n-1} th term to $(2^n - 1)$ th term is 2^{2n-2} .

Since $\{L_n\}$ contains lots of 3, 5, ..., we also can divide them by setting 3, 5, ... be the first term of each group as follows:

$$(1, 1), (3, 1, 5), (3, 7, 1, 9, 5, 11), (3, 13, 7, 15, 1, 17, 9, 19, 5, 21, 11, 23), (3, 25, 13, 27, 7, 29, 15, 31, 1, 33, 17, 35, 9, 37, 19, 39, 5, 41, \dots)$$

$$(1, 1, 3, 1), (5, 3, 7, 1, 9), (5, 11, 3, 13, 7, 15, 1, 17, 9, 19), (5, 21, 11, 23, 3, 25, 13, 27, 7, 29, 15, 31, 1, 33, 17, 35, 9, 37, 19, 39), (5, 41, \dots)$$

We find that if we divide $\{L_n\}$ by 3, the sums of second, third, fourth,...groups are 9,36,144,... respectively; if we divide it by 5, the sums of second, third, fourth,...groups are 25, 100, 400,... respectively;...

Hence, we put forward the second conjecture.

Conjecture 2. The sum of the new sequence $\{L_n\}$ from $k \times 2^{n-1}$ th term to $(k \times 2^n - 1)$ th term is $k^2 \times 2^{2n-2}$, where $k \in N$.

The above transformation is eliminating all the factor 2 for every term of $\{A_n\}$. But what property does the new sequence have if we eliminate all the factor 3 for every term of $\{A_n\}$? Or even more generally, we consider the following question:

Question 1: Let p be a prime. We obtain a new sequence $\{L_n\}$ by transforming $\{A_n\}$ according to the rule $ER(\{p\})$ as follows:

- (1) If A_n has factor p , then we substitute A_n with A_n / p until it does not have factor p ;
- (2) If A_n does not have factor p , then keep it unchanged.

What properties does $\{L_n\}$ have?

The rule $ER(\{p\})$ above can also be written as follows:

If $A_n = p^r \times X$, where $r \geq 0, X \geq 1$ are integers, $(X, p) = 1$, then $L_n = X$.

For this question, we derive the following theorem.

Theorem 1. Let p be a prime. Let $\{L_n\}$ be the sequence generated by transforming $\{A_n\}$ according to the rule $ER(\{p\})$ of Question 1. Then for $k \in N, n \in N$, the following equality holds:

$$\sum_{i=kp^{n-1}}^{kp^n-1} L_i = \frac{(p-1)}{2} k^2 p^{2n-1} \quad (2)$$

Proof. The sum $\sum_{i=kp^{n-1}}^{kp^n-1} L_i$ is from kp^{n-1} th term to $(kp^n - 1)$ th term, which means the number

of the terms is $kp^n - 1 - kp^{n-1} + 1 = k(p-1)p^{n-1}$.

Now, we claim that the $k(p-1)p^{n-1}$ terms are different from each other.

Indeed, assume that there are positive integers i and j such that $kp^{n-1} \leq i < j \leq kp^n - 1$, with $L_i = L_j$. Then by the definition of sequence $\{L_n\}$, we have that $i = p^r \times L_i, j = p^s \times L_j$ where r and s are nonnegative integers. Under the assumption, we have $j = i \times p^{s-r}, s-r \geq 1$, which contradicts with $kp^{n-1} \leq i < j \leq kp^n - 1$.

Furthermore, those $k(p-1)p^{n-1}$ terms are just the positive integers no larger than kp^n except those multiples of p . In fact, the set of positive integers which are no larger than kp^n contains kp^{n-1} multiples of p . Thus, there are $k(p-1)p^{n-1}$ terms left exactly, after these kp^{n-1} multiples of p removed.

Note that the sum of these kp^{n-1} multiples of p removed is $p \times \sum_{i=1}^{kp^{n-1}} i$, so we have

$$\begin{aligned} \sum_{i=kp^{n-1}}^{kp^n-1} L_i &= \sum_{i=1}^{kp^n} i - p \times \sum_{i=1}^{kp^{n-1}} i = \frac{1}{2}(1+kp^n)kp^n - p \times \frac{1}{2}(1+kp^{n-1})kp^{n-1} \\ &= \frac{1}{2}(kp^n)^2 - \frac{1}{2}kp^n kp^{n-1} = \frac{(p-1)}{2}k^2 p^{2n-1} \quad \square \end{aligned}$$

Theorem 2 is derived from Theorem 1.

Theorem 2. Let p be a prime. Let $\{L_n\}$ be the sequence generated by transforming $\{A_n\}$ according to the rule $ER(\{p\})$ of Question 1. Then for $k, n \in \mathbb{N}$, the following equality holds:

$$\sum_{i=1}^{kp^n} L_i = \sum_{i=1}^k L_i + \frac{k^2 p(p^{2n} - 1)}{2(p+1)} \quad (3)$$

Proof. $\sum_{i=1}^{kp^n} L_i = \sum_{i=1}^k L_i + \sum_{j=1}^n \sum_{i=kp^{j-1}+1}^{kp^j} L_i$

$$\begin{aligned}
&= \sum_{i=1}^k L_i + \sum_{j=1}^n \frac{(p-1)}{2} k^2 p^{2j-1} \\
&= \sum_{i=1}^k L_i + \sum_{j=1}^n \frac{1}{2} \left(1 - \frac{1}{p}\right) k^2 p^{2j} \\
&= \sum_{i=1}^k L_i + \frac{1}{2} \left(1 - \frac{1}{p}\right) k^2 \frac{p^2(1-p^{2n})}{1-p^2} \\
&= \sum_{i=1}^k L_i + \frac{k^2 p(p^{2n}-1)}{2(p+1)}. \quad \square
\end{aligned}$$

For the case $k=1$, the following corollary holds:

Corollary 1. Let p be a prime. Let $\{L_n\}$ be the sequence generated by transforming $\{A_n\}$

according to the rule $ER(\{p\})$ of Question 1. Then for $n \in \mathbb{N}$, the following equality holds:

$$\begin{aligned}
(1) \quad &\sum_{i=p^{n-1}}^{p^n-1} L_i = \frac{(p-1)}{2} p^{2n-1} \\
(2) \quad &\sum_{i=1}^{p^n-1} L_i = \frac{p(p^{2n}-1)}{2(p+1)}
\end{aligned}$$

For the case $p=2$, both conjecture 1 and conjecture 2 are correct.

Section 2

In first section, we study the properties of the new sequence generated by eliminating all the factor p in $\{A_n\}$. Naturally, we wish to generalize it for the case of eliminating more primes. In this section, we consider the case of given two primes.

Question 2. Let p and q be primes. We obtain a new sequence $\{H(n)\}$ by transforming $\{A_n\}$ according to the rule $ER(\{p, q\})$ as follows:

(1) If A_n has factor p , then we substitute A_n with A_n / p until it does not have factor

p; If A_n has factor q, then we substitute A_n with A_n/q until it does not have factor q;

(2) If A_n does not have factors p and q, then keep it unchanged.

What properties does $\{H(n)\}$ have?

The rule $ER(\{p, q\})$ above can also be written as follows:

If $A_n = p^r q^s \times X$, where $r \geq 0, s \geq 0, X \geq 1$ are integers, $(X, pq) = 1$, then $H(n) = X$.

For instance, let $p=2, q=3$. Then the generated sequence $\{H(n)\}$ is:

$$1, 1, 1, 1, 5, 1, 7, 1, 1, 5, 11, 1, 13, 7, 5, 1, 17, 1, 19, 5, 7, 11, 23, 1, 25, 13, 1, 7, 29, 5, 31, 1, 11, 17, 35, 1, 37, 1, 41, 43, 47, 53, 59, 61, 65, 71, 77, 83, 89, 95, 101, 107, 113, 119, 125, 131, 137, 143, 149, 155, 161, 167, 173, 179, 185, 191, 197, 203, 209, 215, 221, 227, 233, 239, 245, 251, 257, 263, 269, 275, 281, 287, 293, 299, 305, 311, 317, 323, 329, 335, 341, 347, 353, 359, 365, 371, 377, 383, 389, 395, 401, 407, 413, 419, 425, 431, 437, 443, 449, 455, 461, 467, 473, 479, 485, 491, 497, 503, 509, 515, 521, 527, 533, 539, 545, 551, 557, 563, 569, 575, 581, 587, 593, 599, 605, 611, 617, 623, 629, 635, 641, 647, 653, 659, 665, 671, 677, 683, 689, 695, 701, 707, 713, 719, 725, 731, 737, 743, 749, 755, 761, 767, 773, 779, 785, 791, 797, 803, 809, 815, 821, 827, 833, 839, 845, 851, 857, 863, 869, 875, 881, 887, 893, 899, 905, 911, 917, 923, 929, 935, 941, 947, 953, 959, 965, 971, 977, 983, 989, 995, 1001, 1007, 1013, 1019, 1025, 1031, 1037, 1043, 1049, 1055, 1061, 1067, 1073, 1079, 1085, 1091, 1097, 1103, 1109, 1115, 1121, 1127, 1133, 1139, 1145, 1151, 1157, 1163, 1169, 1175, 1181, 1187, 1193, 1199, 1205, 1211, 1217, 1223, 1229, 1235, 1241, 1247, 1253, 1259, 1265, 1271, 1277, 1283, 1289, 1295, 1301, 1307, 1313, 1319, 1325, 1331, 1337, 1343, 1349, 1355, 1361, 1367, 1373, 1379, 1385, 1391, 1397, 1403, 1409, 1415, 1421, 1427, 1433, 1439, 1445, 1451, 1457, 1463, 1469, 1475, 1481, 1487, 1493, 1499, 1505, 1511, 1517, 1523, 1529, 1535, 1541, 1547, 1553, 1559, 1565, 1571, 1577, 1583, 1589, 1595, 1601, 1607, 1613, 1619, 1625, 1631, 1637, 1643, 1649, 1655, 1661, 1667, 1673, 1679, 1685, 1691, 1697, 1703, 1709, 1715, 1721, 1727, 1733, 1739, 1745, 1751, 1757, 1763, 1769, 1775, 1781, 1787, 1793, 1799, 1805, 1811, 1817, 1823, 1829, 1835, 1841, 1847, 1853, 1859, 1865, 1871, 1877, 1883, 1889, 1895, 1901, 1907, 1913, 1919, 1925, 1931, 1937, 1943, 1949, 1955, 1961, 1967, 1973, 1979, 1985, 1991, 1997, 2003, 2009, 2015, 2021, 2027, 2033, 2039, 2045, 2051, 2057, 2063, 2069, 2075, 2081, 2087, 2093, 2099, 2105, 2111, 2117, 2123, 2129, 2135, 2141, 2147, 2153, 2159, 2165, 2171, 2177, 2183, 2189, 2195, 2201, 2207, 2213, 2219, 2225, 2231, 2237, 2243, 2249, 2255, 2261, 2267, 2273, 2279, 2285, 2291, 2297, 2303, 2309, 2315, 2321, 2327, 2333, 2339, 2345, 2351, 2357, 2363, 2369, 2375, 2381, 2387, 2393, 2399, 2405, 2411, 2417, 2423, 2429, 2435, 2441, 2447, 2453, 2459, 2465, 2471, 2477, 2483, 2489, 2495, 2501, 2507, 2513, 2519, 2525, 2531, 2537, 2543, 2549, 2555, 2561, 2567, 2573, 2579, 2585, 2591, 2597, 2603, 2609, 2615, 2621, 2627, 2633, 2639, 2645, 2651, 2657, 2663, 2669, 2675, 2681, 2687, 2693, 2699, 2705, 2711, 2717, 2723, 2729, 2735, 2741, 2747, 2753, 2759, 2765, 2771, 2777, 2783, 2789, 2795, 2801, 2807, 2813, 2819, 2825, 2831, 2837, 2843, 2849, 2855, 2861, 2867, 2873, 2879, 2885, 2891, 2897, 2903, 2909, 2915, 2921, 2927, 2933, 2939, 2945, 2951, 2957, 2963, 2969, 2975, 2981, 2987, 2993, 2999, 3005, 3011, 3017, 3023, 3029, 3035, 3041, 3047, 3053, 3059, 3065, 3071, 3077, 3083, 3089, 3095, 3101, 3107, 3113, 3119, 3125, 3131, 3137, 3143, 3149, 3155, 3161, 3167, 3173, 3179, 3185, 3191, 3197, 3203, 3209, 3215, 3221, 3227, 3233, 3239, 3245, 3251, 3257, 3263, 3269, 3275, 3281, 3287, 3293, 3299, 3305, 3311, 3317, 3323, 3329, 3335, 3341, 3347, 3353, 3359, 3365, 3371, 3377, 3383, 3389, 3395, 3401, 3407, 3413, 3419, 3425, 3431, 3437, 3443, 3449, 3455, 3461, 3467, 3473, 3479, 3485, 3491, 3497, 3503, 3509, 3515, 3521, 3527, 3533, 3539, 3545, 3551, 3557, 3563, 3569, 3575, 3581, 3587, 3593, 3599, 3605, 3611, 3617, 3623, 3629, 3635, 3641, 3647, 3653, 3659, 3665, 3671, 3677, 3683, 3689, 3695, 3701, 3707, 3713, 3719, 3725, 3731, 3737, 3743, 3749, 3755, 3761, 3767, 3773, 3779, 3785, 3791, 3797, 3803, 3809, 3815, 3821, 3827, 3833, 3839, 3845, 3851, 3857, 3863, 3869, 3875, 3881, 3887, 3893, 3899, 3905, 3911, 3917, 3923, 3929, 3935, 3941, 3947, 3953, 3959, 3965, 3971, 3977, 3983, 3989, 3995, 4001, 4007, 4013, 4019, 4025, 4031, 4037, 4043, 4049, 4055, 4061, 4067, 4073, 4079, 4085, 4091, 4097, 4103, 4109, 4115, 4121, 4127, 4133, 4139, 4145, 4151, 4157, 4163, 4169, 4175, 4181, 4187, 4193, 4199, 4205, 4211, 4217, 4223, 4229, 4235, 4241, 4247, 4253, 4259, 4265, 4271, 4277, 4283, 4289, 4295, 4301, 4307, 4313, 4319, 4325, 4331, 4337, 4343, 4349, 4355, 4361, 4367, 4373, 4379, 4385, 4391, 4397, 4403, 4409, 4415, 4421, 4427, 4433, 4439, 4445, 4451, 4457, 4463, 4469, 4475, 4481, 4487, 4493, 4499, 4505, 4511, 4517, 4523, 4529, 4535, 4541, 4547, 4553, 4559, 4565, 4571, 4577, 4583, 4589, 4595, 4601, 4607, 4613, 4619, 4625, 4631, 4637, 4643, 4649, 4655, 4661, 4667, 4673, 4679, 4685, 4691, 4697, 4703, 4709, 4715, 4721, 4727, 4733, 4739, 4745, 4751, 4757, 4763, 4769, 4775, 4781, 4787, 4793, 4799, 4805, 4811, 4817, 4823, 4829, 4835, 4841, 4847, 4853, 4859, 4865, 4871, 4877, 4883, 4889, 4895, 4901, 4907, 4913, 4919, 4925, 4931, 4937, 4943, 4949, 4955, 4961, 4967, 4973, 4979, 4985, 4991, 4997, 5003, 5009, 5015, 5021, 5027, 5033, 5039, 5045, 5051, 5057, 5063, 5069, 5075, 5081, 5087, 5093, 5099, 5105, 5111, 5117, 5123, 5129, 5135, 5141, 5147, 5153, 5159, 5165, 5171, 5177, 5183, 5189, 5195, 5201, 5207, 5213, 5219, 5225, 5231, 5237, 5243, 5249, 5255, 5261, 5267, 5273, 5279, 5285, 5291, 5297, 5303, 5309, 5315, 5321, 5327, 5333, 5339, 5345, 5351, 5357, 5363, 5369, 5375, 5381, 5387, 5393, 5399, 5405, 5411, 5417, 5423, 5429, 5435, 5441, 5447, 5453, 5459, 5465, 5471, 5477, 5483, 5489, 5495, 5501, 5507, 5513, 5519, 5525, 5531, 5537, 5543, 5549, 5555, 5561, 5567, 5573, 5579, 5585, 5591, 5597, 5603, 5609, 5615, 5621, 5627, 5633, 5639, 5645, 5651, 5657, 5663, 5669, 5675, 5681, 5687, 5693, 5699, 5705, 5711, 5717, 5723, 5729, 5735, 5741, 5747, 5753, 5759, 5765, 5771, 5777, 5783, 5789, 5795, 5801, 5807, 5813, 5819, 5825, 5831, 5837, 5843, 5849, 5855, 5861, 5867, 5873, 5879, 5885, 5891, 5897, 5903, 5909, 5915, 5921, 5927, 5933, 5939, 5945, 5951, 5957, 5963, 5969, 5975, 5981, 5987, 5993, 5999, 6005, 6011, 6017, 6023, 6029, 6035, 6041, 6047, 6053, 6059, 6065, 6071, 6077, 6083, 6089, 6095, 6101, 6107, 6113, 6119, 6125, 6131, 6137, 6143, 6149, 6155, 6161, 6167, 6173, 6179, 6185, 6191, 6197, 6203, 6209, 6215, 6221, 6227, 6233, 6239, 6245, 6251, 6257, 6263, 6269, 6275, 6281, 6287, 6293, 6299, 6305, 6311, 6317, 6323, 6329, 6335, 6341, 6347, 6353, 6359, 6365, 6371, 6377, 6383, 6389, 6395, 6401, 6407, 6413, 6419, 6425, 6431, 6437, 6443, 6449, 6455, 6461, 6467, 6473, 6479, 6485, 6491, 6497, 6503, 6509, 6515, 6521, 6527, 6533, 6539, 6545, 6551, 6557, 6563, 6569, 6575, 6581, 6587, 6593, 6599, 6605, 6611, 6617, 6623, 6629, 6635, 6641, 6647, 6653, 6659, 6665, 6671, 6677, 6683, 6689, 6695, 6701, 6707, 6713, 6719, 6725, 6731, 6737, 6743, 6749, 6755, 6761, 6767, 6773, 6779, 6785, 6791, 6797, 6803, 6809, 6815, 6821, 6827, 6833, 6839, 6845, 6851, 6857, 6863, 6869, 6875, 6881, 6887, 6893, 6899, 6905, 6911, 6917, 6923, 6929, 6935, 6941, 6947, 6953, 6959, 6965, 6971, 6977, 6983, 6989, 6995, 7001, 7007, 7013, 7019, 7025, 7031, 7037, 7043, 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