# Error-Bounded Surface Remeshing with Minimal Angle Elimination

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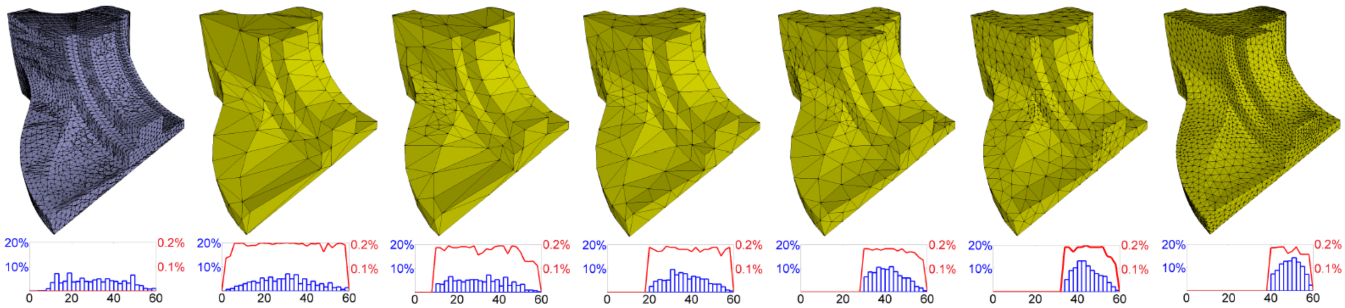


(a) Input model. (b) $\theta = 0°$. (c) $\theta = 10°$. (d) $\theta = 20°$. (e) $\theta = 30°$. (f) $\theta = 35°$. (g) $\theta = 40°$.

**Figure 1:** *The input (a) and the results ((b) to (g)) with different minimal angle threshold $\theta$. The angle distributions and their approximation error are shown below, in blue bar and red curves respectively. The error bound $\delta$ is set to $0.2\%$ of the input's bounding box (%bb).*

## Abstract

Surface remeshing is a key component in many geometry processing applications. However, existing high quality remeshing methods usually introduce approximation errors that are difficult to control, while error-driven approaches pay little attention to the meshing quality. Moreover, neither of those approaches can guarantee the minimal angle bound in resulting meshes. We propose a novel error-bounded surface remeshing approach that is based on minimal angle elimination. Our method employs a dynamic priority queue that first parameterize triangles who contain angles smaller than a user-specified threshold. Then, those small angles are eliminated by applying several local operators ingeniously. To control the geometric fidelity where local operators are applied, an efficient local error measure scheme is proposed and integrated in our remeshing framework. The initial results show that the proposed approach is able to bound the geometric fidelity strictly, while the minimal angles of the results can be eliminated to be up to 40 degrees.

**Keywords:** surface remeshing, error-bounded, minimal angle

**Concepts:** •**Computing methodologies** → **Computer graphics;** *Mesh geometry models;*

## 1 Introduction and Motivation

Surface remeshing is a key component in many geometry processing applications. While many remeshing techniques are goal-specified, a common goal of most of them is to find a compromise among the following three aspects: (a) **Geometric fidelity**, which is usually measured as approximation error, is the key requirement for most applications; (b) **Element quality**. The quality of the mesh elements is crucial for numerical stability, which require fairly regular meshes in terms of both geometry and connectivity. Particularly, a lower bound on the minimal angle is vital for many numerical simulations; and (c) **Mesh complexity**. Efficient representation of complex shapes is of fundamental importance. Since mesh complexity usually conflicts with geometric fidelity and elements quality, the "just enough" resolution for the required elements quality and geometric fidelity should be the goal.

However, to the best of our knowledge, most existing methods that generate meshes with high elements quality often require high mesh complexity or introduce high approximation error. The error-driven methods, while preserving the results in controllable geometric fidelity and low mesh complexity, pay little attention to the elements quality. To surmount the above limitations, we propose an error-bounded surface remeshing method based on minimal angle elimination. Our method requires the user only to specify the approximation error bound threshold (i.e., Hausdorff distance) and the desired minimal angle. It then eliminates the minimal angle of the input mesh up to the specified minimal angle threshold, with the constraint of the specified error bound.

Contrary to the existing methods that iteratively apply the local operators sequentially and globally [Alliez et al. 2008], we employ a dynamic priority queue to parameterize all triangles that contain angles smaller than the user specified threshold, and then locally apply these operators. Since our method only improves input meshes in local regions where the elements quality is poor, it modifies the models as little as possible with respect to the minimal angle threshold and the error-bound constraint.

## 2 Technical Approach

Given an input 2-manifold triangular mesh $M_I$, the minimal angle threshold $\theta$ and the error bound $\delta$, the output result $M_R$ is first initialized as a copy of $M_I$. Next, we parameterize all the small angles in $M_R$ by filling their opposite halfedges into a dynamic priority queue $Q$. Each time when a halfedges $h$ is removed from $Q$, we sequentially check whether the trigger conditions of collapsing $h$ or relocating the vertices of the facet that contains $h$ can be satisfied. If so, the according local operator is applied; otherwise, the

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SIGGRAPH '16, July 24-28, 2016, Anaheim, CA,
ISBN: 978-1-4503-4371-8/16/07
DOI: http://dx.doi.org/10.1145/2945078.2945138

*Longest-Side Propagation Path* [Rivara 1996] algorithm is applied to find the local longest edge $h_l$ on $M_R$, and then apply the edge split on $h_l$. Each time when a local operator is applied, we optimize the position(s) of the generated or affected vertex(es) for geometric fidelity reduction. Finally, $Q$ is updated for the next iteration. The algorithm terminates when $Q$ is empty.

### 2.1 Local Error Measure Scheme

A reliable error measure scheme is necessary to bound the geometric fidelity between $M_R$ and $M_I$. Since each local operator only modifies a local area of $M_R$, we design an efficient local error measure scheme based on the approximated Hausdorff distance, and embed it into the trigger conditions of local operators.

To improve the efficiency, we use the limited samples to approximate the exact Hausdorff distance between $M_R$ and $M_I$. Suppose the sample sets on $M_I$ and $M_R$ are $S_I$ and $S_R$, respectively, then the Hausdorff distance between $M_I$ and $M_R$ can be approximated as $d_H(M_I, M_R) \approx max\left(max_{a\in S_I} d(a,\hat{a}), max_{b\in S_R} d(b,\hat{b})\right)$, where $\hat{a}$ is the closest point to $a$ on $M_R$, and $\hat{b}$ is the closest point to $b$ on $M_I$. To better measure the approximation error on features, we evenly sample facets, edges and vertices of $M_I$ and $M_R$.

In order to bound the error between $M_R$ and $M_I$, we have to trace the approximation error each time when a local operator is to be applied. However, maintaining the global closest points of the samples is time-consuming, since each time when a local operator is applied, the searching data structure of $M_R$ has to be rebuilt. Based on the observation that each local operator only modifies the geometry and the topology of a small patch on $M_R$, we propose a local error measure scheme based on local closest points update. The key idea is that each time when a local operator is applied, only the closest point pairs in the *local affected patches* are updated. Hence, instead of searching the global closest points of the samples from $M_I$ on the whole $M_I$, we only search the closest points in a local patch of $M_R$. Since the local data structure contains very few elements and is easy to build, the proposed scheme is much more efficient than the global error measure scheme, yet provides a reliable upper error bound between $M_R$ and $M_I$.

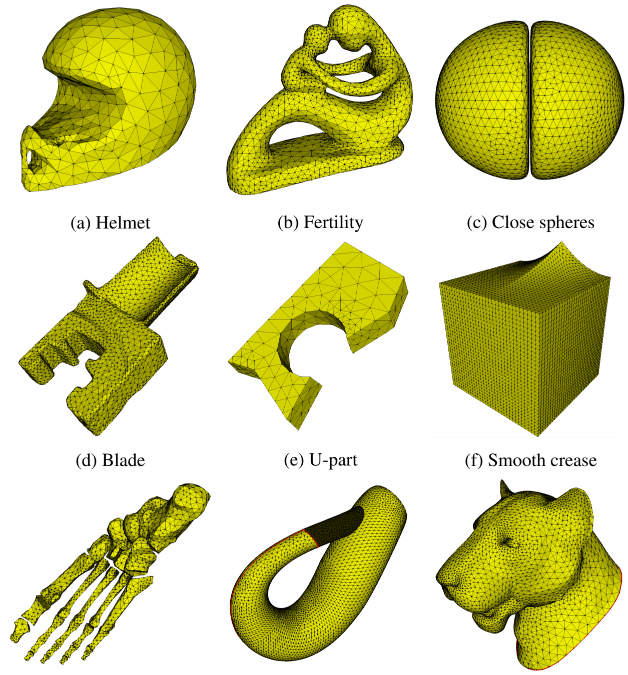
### 2.2 Trigger Conditions of Local Operators

We employ three basic local operators, namely edge split, edge collapse and vertex relocate. As edge split operator neither alters the topology of the mesh nor introduces approximation error, its trigger condition is always specified as true. We only define the trigger conditions of edge collapse and vertex relocate that helps to maintain the geometric fidelity and topological invariance of $M_R$ while improving its elements quality. Generally, there are four necessary constraints for specifying the trigger conditions of local operators:

- **Topology constraint**. The local operator should always guarantee that $M_R$ and $M_I$ are isotopic [Edelsbrunner 2006];
- **Geometry constraint**. The local operator should not flip the incident 1-ring facets with respect to the current 1-ring facets;
- **Fidelity constraint**. The Hausdorff distance between $M_R$ and $M_I$ should stay below $\delta$ when a local operator is applied;
- **Quality constraint**. The local operator should not introduce new angles smaller than the current minimal angle.

We declare that the edge collapse can be applied if and only if when all the four constraints are satisfied. For vertex relocate, we only require the last three constraints since this kind of local operator does not change the topology of $M_R$.

## 3 Results and Future work



(a) Helmet (b) Fertility (c) Close spheres (d) Blade (e) U-part (f) Smooth crease (g) Bones (h) Klein's bottle (i) Lion head

**Figure 2:** *Some selected results. Here the parameter $\delta$ is set to $0.2\%(bb)$ and $\theta$ is set to $35°$ in all inputs. In the Klein bottle and Lion head models, the boundaries are visualized in red. The dark part of the Klein bottle means the triangle normals are inside.*

We have implemented a primary prototype of our algorithm in C++, and tested some representative models in a 64-bit Windows 8.1 operating system. See Fig. 2 for some selected examples. The results show that the proposed approach bounds the geometric fidelity strictly and works well for models with different characteristics. Since we can guarantee the minimal angle of the results, we believe our methods will be more suitable in applications such as numeric simulation and robust geometry processing.

In the future, we plan to investigate the theoretical and experimental convergence of our algorithm. We would also like to make a thorough comparison between our results and the state-of-the-art surface remeshing approaches in terms of geometric fidelity, elements quality and mesh complexity.

## Acknowledgements

This work was partially supported by the National Natural Science Foundation of China (Nos. 61373071, 61372168 and 61271431).

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