Evolution, prime numbers, and an algorithm for the creative process

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Key words: evolution, Darwinism, molecular clock, neutral theory, MGD theory, prime numbers, Riemann Hypothesis, uniqueness, uniformity, creativity, the Prime Law

Abstract

Recent understandings of molecular evolution, together with the fossil records, have established that there are both linear and nonlinear processes in the creation of novel species, which is strikingly similar to the generation of prime numbers and human creativity. Each creation of a more complex species is like a prime number, unpredictable, discontinuous, and yet can be modeled by a smooth curve in relation to time. The mystery behind the complexity increases in nature and human civilizations might well turn out to be similar to that behind the appearances of prime numbers. Here we show that an algorithm for the creative process of humans can create prime numbers in a lawful and yet unpredictable fashion. The essence of primes is the duality of uniqueness and uniformity together with the creation algorithm. The algorithm consists of the non-linear process of uniformity selection to create the unique and the linear process of uniqueness selection to form the uniformity. The iterations of this algorithm can create an infinite number of primes. The algorithm appears to have been hardwired in the human brain as shown by recent experimental studies. This new understanding can deduce some of the best-known properties of primes and may explain the nearly constant and yet seemingly random creation of novelty in relation to time.

Introduction:

Evolution and evolutionary theories

To use the concept of evolution to explain the seemingly endless creations of species in life history was largely made popular by the works of Darwin and Wallace. The concept was inspired by observing the phenotypic adaptations of living species as well as the fossil records. Phenotypic variations were thought to be randomly generated, which are followed by natural selection to either keep the fit or eliminate the unfit variants. While many have argued that natural selection is not a random process, no one has disputed that the appearance of an environmental condition, such as a hot weather, is anything but random. A single random event is enough to make the outcome of a chain of events random. The selector in the natural selection processes is always random in a Darwinian intention-less world. The natural selection process from the selector hot weather to the elimination of heat sensitive variants is surely non-random. But the selector hot weather is random, which makes the end results of natural selection random. Calling the process non-random is not wrong but is meaningless. The end result is what matters, which can only be either random or intentional. The natural selection process is non-random but the end result could still be random if the selector is randomly caused. Thus, the creation of novelty as explained by Darwin and Wallace is strictly a linear process of creation by chances or accidents.

The chance creation theory of Darwin and Wallace, while does explain well microevolution or small scale changes such as the appearance of drug resistance in bacteria, has long found difficulty, or has not met with any evidence, in explaining macroevolution or large scale changes in complexity such as the formation of the first life from inorganic materials or the advance from single cell organisms to multicellular species. Furthermore, the theory has been challenged by the most astonishing phenomenon in evolution at the level of genomes or molecules as first discovered in the early 1960s, the genetic equidistance phenomenon [1-3]. For any three or more species of different organismal complexity as intuitively defined by the number of cell types, one can perform two kinds of sequence alignment. The first aligns a complex organism such as human against simpler or less complex species that evolved earlier such as frogs and fishes. The second aligns simpler organisms such as fishes against those more complex ones such as chickens and humans. The

first kind of alignment shows the pattern that human shares more identity with chickens than with fishes or a hierarchy of increasingly less identity to increasingly less complex species, which is largely consistent with Darwinian expectations [1]. Margoliash in 1963 performed both alignments and made a formal statement of the molecular clock hypothesis after noticing the genetic equidistance result where fishes are equidistant to chickens and humans or simple species are equidistant to all more complex species, which is unexpected from Darwin's theory [2, 4]. In hindsight, however, Margoliash has mistakenly converted a maximum saturation phenomenon that can vary across species and populations into a linear-phase phenomenon.

The nearly constant and similar mutation rate (i.e., molecular clock) interpretation of the equidistance phenomenon has in fact turned out to be a classic tautology since it has not been verified by any independent observation and has on the contrary been contradicted by a large number of facts [5-10]. Nonetheless, researchers had initially treated the molecular clock as a genuine reality and had in turn proposed a number of theories to explain it [11-16]. The 'Neutral Theory' has become the favorite [14-16], even though it is widely acknowledged to be an incomplete explanation for the clock [9, 17]. The observed rate is measured in years but the Neutral theory predicts a constant rate per generation. Also, the theory predicts that the clock will be a Poisson process, with equal mean and variance of mutation rate. Experimental data have shown that the variance is typically larger than the mean. Ohta's "nearly neutral theory" explained to some extent the generation time issue by observing that large populations have faster generation times and faster mutation rates but remains unable to account for the great variance issue [18]. With the neutral and nearly neutral theory, molecular evolution has been treated as the same as population genetics or microevolution. However, the field still lacks a complete theory as many have acknowledged [19, 20], and has unfortunately yet to pay attention to the equidistance result, which has been considered by some as "one of the most astonishing findings of modern science"[21, 22].

We recently proposed the maximum genetic diversity (MGD) hypothesis to explain the genetic equidistance phenomenon based on a pair of intuitions or axioms [8, 23-25]. Axiom 1 posits that the more complex the phenotype, the greater the restriction on the choices or errors in molecular building parts. Axiom 2 says that any system can allow a

limited level of random errors or noises in molecular building parts and such errors may be beneficial, deleterious, or neutral depending on circumstances. Obviously, one only needs to substitute "errors in molecular building parts" for "genetic diversity" to get the equivalent concepts in biology. Axiom 2 in effect underlies the proven virtues of the modern evolution theory consisting of Darwin's and Kimura's theories. It is because species have built-in robustness or allowed range of random errors in the first place that chance events could lead to limited order. If an organism is built in a way that cannot allow any random errors in its genome, chance would be only destructive and Darwinian mechanisms would not be able to work at all to create order.

Genetic diversity or distance cannot increase indefinitely with time and has a maximum limit being restricted by function and physiology or epigenetic complexity. The MGD of simple organisms is greater than that of complex organisms. Over long evolutionary time, the genetic distance between sister species and a simpler outgroup (more distant) taxon is mainly determined by the MGD of the simpler outgroup, although over short time scales it is determined by time, drift, environmental selection, and the neutral mutation rates of the simpler outgroup as well as to a smaller extent by the rates of the sister taxa. The MGD hypothesis thus includes the proven virtues of modern evolution theory, consisting of Darwin's theory and the neutral theory, as relevant only to microevolution over short time scales before sequence divergence reaches MGD. An increase in epigenetic complexity during macroevolution is associated with a suppression of genetic diversity or point mutations. So, evolution involves two distinct processes, linear or microevolution versus nonlinear or macroevolution (Fig. 1). The linear process is largely just accumulating random mutations followed by drift or natural selection. The nonlinear process is a sudden and dramatic change or increase in the complexity of epigenetic programs that necessarily demands a reduction in MGD or the maximum tolerable level of random errors in the genomes. The timing for the epigenetic change is not arbitrarily or randomly set but is determined by the time required for the prior species to reach MGD or maximum mutation saturations. The origin of life from inorganic materials can be likened as a reduction in entropy as life building molecules must lose degree of freedom when turning from its inorganic existence into existence in a life form. According to the MGD theory, such entropy reduction underlies not only the origin of the

first life but also the origin of each and every major advance in complexity in the macroevolution of species.

The MGD hypothesis explains the genetic equidistance phenomenon as a result of maximum genetic distance imposed by physiological or epigenetic constraints [8, 23-25]. This phenomenon has in fact another characteristic, the overlap feature where particular sites in an amino acid sequence are subject to multiple different mutational changes in a particular lineage which has been overlooked for nearly half of a century [26]. While the molecular clock may superficially explain the apparent equidistance in quantities, it cannot explain the non-random distribution of mutation hot spots and the related observation that the percentage of constrained sites in more complex clades is greater than that in simpler organisms. The MGD theory has accounted for major phenomenology of molecular evolution. It has also been instrumental in directing productive research into not only evolutionary phylogenetic problems but also key biomedical problems [26-37].

In a sequence alignment with humans, there is a hierarchy with humans less and less related to increasingly less complex species (Fig. 2). As less complex species evolved earlier, the hierarchy of gene identities shows correlations with two different parameters, complexity and time. If one only focused on the time correlation, one would conclude that protein non-identity is only determined by time of separation as if the substitution rate is constant and the same among species (hence the molecular clock). On the other hand, if one focused on the complexity parameter and ignored time, one would find a strong correlation of sequence identity with species complexity. One also finds that simple species is equidistant to all more complex species. So, the distance hierarchy with humans as measured by fast evolving proteins at maximum saturation distance is a result of lower and lower complexity of species in more ancient times and hence increasingly higher within-species MGD. The saturation distance to human for a lower complexity species is equal to the within-species MGD of the lower taxon.

Genomes have two types of sequence mismatches, functional and neutral, both of which show correlation with time. The neutral variations are explained by the neutral theory and show correlation with time during the linear phase of evolution. The functional variations are correlated with physiology, as explained by the MGD theory, and indirectly with time as

the degrees of physiological complexity are correlated with time with simple physiology evolved earlier in time.

The molecular clock or constant rate interpretation is really about the nearly constant rate of complexity increases. People since Aristotle have long appreciated the direction of evolution towards higher complexity. Scholars believing in Darwin's theory have always denied this but only by ignoring or misreading inconvenient facts such as the genetic equidistance phenomenon. The evidence for complexity increases is commonplace and easy to notice. The first molecular evidence for it is the maximum genetic equidistance phenomenon. What is most striking is the nearly constant rate of complexity increases as measured in years, which can be quantitatively shown by the fraction of non-changeable positions in a protein or the fraction of identical residues between human and a lower complexity species (Fig. 2). Such nearly constant rate is inconsistent with chance creations as chance would mean much greater irregularity: events may cluster at some time windows while rarely happen at some other time windows. The challenge is how to prove that it is not chance.

Prime numbers

As nature is written in the language of mathematics, it would be most unusual if a fundamental natural phenomenon, i.e., the nearly constant rate of evolution towards higher complexity as measured in years (Fig. 2), has no counterpart in mathematics and vice versa. An intriguing analogy is the pattern of prime numbers [38]. The cumulative increase in prime numbers along the progression in natural numbers is well known to follow a smooth curve with a nearly constant rate, especially when viewed from a distance (Fig. 3) [39, 40]. Here the progression in natural numbers is like a time clock, rigid and predictable. The appearance of prime numbers is discontinuous like a staircase and unpredictable but follows nonetheless a well-defined function Li(N) as shown by the Riemann hypothesis [38]. Such a pattern is inconsistent with chance as chance would mean much greater irregularity. Each new appearance of a more complex species is like a new prime number, unpredictable, discontinuous, and yet nearly constant. Individual species are well known to appear in the fossil record abruptly as evidence for the punctuated equilibrium model of macroevolution

has shown [41]. However, the discontinuous appearance of species of higher and higher complexity still follows a very smooth and regular pattern as shown by the equidistance phenomenon (Fig. 2). The striking similarity between evolution and prime numbers suggests that there may be a common explanation underlying both.

The German mathematician Bernhard Riemann formulated the Riemann Hypothesis (RH) in 1859 [42]. The hypothesis is widely regarded as the most important unsolved problem in all of mathematics. The RH is believed by most mathematicians to be true. A large number of deep and important other results have been proven under the condition that it holds. The RH essentially says that the primes are as regularly distributed as possible given their seemingly unpredictable occurrence on the number line. According to the Prime Number Theorem of Gauss, the number of primes less than N is approximately the logarithmic integral Li(N) or less precisely N/ln(N). If the RH is true, the error between Li(N) and the true number of primes is at most of the order of the square root of N [38, 40, 43-45]. This error margin is the smallest possible and cannot be improved by much [46]. This is the error margin expected by the theory of probability for some unpredictable events such as a coin toss. Random means no pattern and yet mathematics is largely about finding and proving patterns. The difficulty with the RH probably lies in the fact that it requires one to prove the absence of a pattern, which has rarely if ever been done in mathematics. How does one demonstrate unpredictable or “absence of a pattern” as wondered by Tao [47]? Is it even possible to do?

A prime number is commonly defined as a positive integer that has only two divisors, 1 and itself. Both the number 1 and 2 can be either included or excluded as primes by manipulating the definition of primes. Accordingly, the primality of the number 1 and 2 are decided by human agreements rather than objective logic or reason. The number 1 is not considered a prime today but was in the past [48-52]. While 2 is considered a prime today, at one time it was not [53]. The odd primes have many properties not shared by 2, the only even prime. It is also easy to have a definition based on calculation that would include all primes except 2. Thus, a prime can be defined as a positive integer that cannot be expressed by the even number of sums of any single number except 1 and itself. For example, 1 is 0 (an even number) sum of 1 and itself; 3 is 2 sums of 1 and 0 sum of itself; but 2 is not a prime since it

is 1 (an odd number) sum of 1.

To define numbers by calculation that is itself defined by numbers is a tautology, which merely describes ways of identifying some primes but reveals little about what a prime really is or the essence of primes. A tautological or circular definition necessarily means a lack of true understanding. This leads to the dilemma that 2 is a prime in one definition based on division but not a prime in another equally plausible definition based on addition. It is arbitrary human convenience or taste to favor one tautological definition over another. We can only resolve such dilemma with objective reasoning when we achieve a deeper understanding of primes that is based on knowledge more fundamental than calculation and numbers. Primes are the foundations of mathematics and should have a form of existence or definition that is independent of mathematics.

To avoid circularity, a creature must be defined by things that are more basic than it rather than more advanced. We must use quantum particles rather than molecules to define atoms, even though we discovered molecules before we knew about quanta. Just because calculation was discovered before prime numbers in human history does not mean that primes must be defined by calculation. A concept can only be defined by concepts lower or more basic in logical hierarchy. What is even more basic than numbers must be used to define primes and non-primes. If primes are atoms that build other numbers, then the primes must be built by its own building blocks, which would be equivalent to quantum particles.

The essence of a creature is its building blocks together with a rule of manipulating the building blocks. The essence is what is ultimately responsible for the properties of a creature. The essence of matter, the quanta building block together with a law of manipulating the quanta, is what is ultimately responsible for the properties of the physical universe. A creature must be defined by its essence. Numbers are creatures of the mind. Are primes related to the creativity of the mind?

The creative process of the mind

The creativity of the human mind is the most remarkable feature of humans that sets humans apart from all other biological species. Comparing today's civilization with those of a few thousand years ago, it is clear that humans have been constantly creating things, both

physical and metaphysical. While the history of human civilization has seen a countless number of human creations, with the recent creations generally more complex than earlier ones, it seems that the basic capacities of the human brain have remained relatively unchanged at least within the last 5000 years. It seems to be a real phenomenon that more complex things get created over time while the basic capacities of the brain have stayed largely the same. The brain appears to have the ability to know or absorb whatever that have been created before and to come up with novel inventions.

There is likely a general pattern or rule that can describe the creative phenomenon of the mind. While creations can be countless and different, the general rule or algorithm employed by the mind for each creation may be the same. Creations by successive generations may be viewed as the iterative applications of the same creation algorithm. To discover that algorithm may be important in order to understand the structures and functions of the brain that seem to be able to absorb past creations and to come up with something new. It may also help to reveal why the mind always becomes bored with new things after a while and why it has an insatiable appetite for novelty.

It has not escaped attention by scholars that evolution of species and creative evolution in human civilizations share similarities. The generation of creative ideas is generally viewed as an evolutionary process. Some think it is Darwinian [54, 55], while others not [56]. The question addressed by those studies is how a creative idea evolves from a population of competing ideas within a mind and how insights from evolution of species may help model human creations. Few if any, however, has attempted to independently come up with an algorithm or understanding of the creative evolutionary process of the mind and see how well it may also describe the evolutionary history of species. The question that concerns us here is at a fundamental level: what motivates a mind to create in the first place. The creative process that we would like to analyze is akin to the paradigm evolution process of Kuhn [57]. A paradigm is often initiated by a creative individual and gets established subsequently by countless individuals who make incremental advances within the paradigm. The former is a nonlinear process while the latter linear. For an established paradigm nearing maximum saturation in terms addressing details, some begin to see some major problems while most do not. Then a creative individual comes up with a revolutionary solution to a

major problem of the established paradigm and a new paradigm is initiated. The creative process from paradigm to paradigm continues seemingly without end. Here, we aim to develop a general hypothesis of the creative process underlying human creative activities.

The creation algorithm that programs the mind to be creative is obviously the foundation of all human creations. Since numbers (positive integers) may represent the most fundamental creatures of the mind, the creation algorithm of the mind should be able to create numbers. We here found that prime numbers can model the creative process of the mind. Others have independently noticed the connection between prime numbers and creation [58]. Louis Kauffman and Hector Sabelli observed: "The generation of primes epitomizes the causal creation of novelty." [59]. Don Zagier noted: "Upon looking at these numbers, one has the feeling of being in the presence of one of the inexplicable secrets of creation." [60].

If creation by the mind is lawful or deterministic, i.e., determined by an algorithm, it could be predictable. However, if creation is predictable, it would no longer by definition be novel or unique. For creation to be meaningful to humans, it must not be predictable. Human creativity is also logical and reasoned and does not seem to be arbitrary. Thus, whatever the algorithm that has programmed the human mind and made the mind creative must make the creative process lawful and yet the outcome unpredictable. Similarly, prime numbers have been found to be both lawful and seemingly unpredictable. But a deterministic law of primes or of human creativity remains to be discovered that nonetheless cannot predict the outcomes.

Primes are both lawful and seemingly unpredictable. This is highly similar to human creations and appearances of increasingly complex species in macroevolution. Here, we first describe a creation algorithm that makes the creative process lawful but the outcome unpredictable. We then show that this algorithm can create primes in a fully deterministic and lawful process but still allows the primes to have the intrinsic property of unpredictability. The implications of this algorithm to RH, creativity, and evolution in nature are then discussed.

Results and Discussion:

The algorithm that makes the mind creative

The most fundamental capacities of a human mind may be to know and to imagine, which are essential to creativity. To know is to recognize the unique from a background of contrasting uniformity and vice versa. To imagine is to think of novel things that do not exist previously. By observing how the human mind creates, we have found an algorithm that programs the mind and makes the mind creative. This algorithm consists of a pair of opposite but complimentary *yin* and *yang* principles with one underlying a linear process and the other a nonlinear process, and a mind that coordinates the interplay of the two principles. A creation or creature is defined as the unique that does not exist previously, is distinguished from all other imagined things, and can exist subsequent to its creation by being able to initiate a population of followers that share a uniform pattern resembling the unique. A creation has the bipolar duality of uniqueness and uniformity. A follower of a creation is defined as the new thing that does not exist previously but shares some uniform property with a prior creation. A creation is a large advance in paradigm while a follower of a creation represents a small step progress within a paradigm.

The imagination of a mind is either following the existing patterns of past creations or is based on a novel pattern. How does a novel pattern come to the mind remains a mystery and is of no concern here. A new but meaningless thing or pattern is not a creation because it cannot be uniquely distinguished from other imagined entities, cannot be logically linked with existing patterns, and cannot initiate a following. A great piece of music or book or art initiates a following by existing in the minds of people who are familiar with the piece. A book that was soon forgotten forever is not a creation but is merely a follower of an existing pattern. Existing pattern consists of both past creations and of a default order-less state. The order-less state is the background and driving force for order and pattern. A new thing that is not following any existing ordered pattern but is not uniquely distinguished from the order-less state is still viewed as a follower because it is following the existing order-less pattern. Things that constitute the order-less state include all that cannot be logically linked to any ordered pattern and cannot be uniquely distinguished from others or are equally unique as others.

The *yang* principle for the nonlinear process is uniformity selection that allows the mind to recognize the unique or the creation. Uniformity abolishes individuality and selects

for the unique. Uniformity selection drives the creation of the unique. The *yin* principle for the linear process is uniqueness selection that allows the unique to initiate a population of followers sharing a uniformity pattern resembling the unique. The mind uses this principle to allow the unique to exist or survive subsequent to its creation. Uniqueness selection results in the formation of an ordered uniformity consisting of individuals that are fittest or most adapted to the unique. The process from the unique to a specific uniformity of a population of followers is essential for the unique to exist subsequent to its creation, which further serves to drive the creation of the next unique. The creative process of the mind is the iterative use of the same creation algorithm and an endless cycling process from uniformity to unique to new-uniformity. When the mind sees the unique, the mind strives to fit and follow. When the mind sees uniformity, the mind strives to be unique. All human minds are a unity of different degrees of the *yin* and *yang* principles. The nonlinear process requires more work from the mind than the linear process.

To create, the mind needs to know what is known previously, which is termed the existing-uniformity. Selection by existing-uniformity allows the mind to know whether something is new with a meaningfully ordered pattern. In addition, all creations begin from the imagination of the mind. Within the imagined world, there exists a unique entity that is distinguished from the imagined-uniformity shared by other imagined entities. To create by uniformity selection is to bring into existence an imagined entity that is distinct from both the existing-uniformity and the imagined-uniformity.

The formation of the order-less existing uniformity is by the default of reproduction and the inherent nature of the mind. The mind treats anything that cannot be rationalized with past creations as part of an order-less uniformity. Random brushes on a canvas would belong to the order-less uniformity. The formation of the ordered existing-uniformity requires the principle of uniqueness selection. This selection process selects individuals to follow the unique creation of the past. The followers of a unique creation are essential to the popularization of the unique and the long-term existence of the unique in the form of existing-uniformity. The followers also contribute new variations or incremental advances around the main theme/paradigm of the unique creation, which would form a new level of existing-uniformity essential for triggering the next unique creation. However, the

incremental progress made by the followers cannot directly in itself lead to the next unique creation. Creation of the unique represents a discontinuous nonlinear change in paradigm and is fundamentally different from the linear formation of followers.

Existing uniformity thus consists of order-less and ordered. Based on the existing uniformity, the mind is able to know whether something is imagined or not yet existing. Among the things imagined, a uniform property may be shared by all except the unique. The unique is the one that has the closest relationship to the existing uniformity but does not belong to any of the existing paradigms. The creation of the unique cannot come as a logical extension of an existing pattern but is nevertheless logically related to existing patterns after the fact of creation.

Creating primes by the creation algorithm

Like creations of the mind, the odd primes including the number 1 also have the dual property of uniqueness and uniformity. A thing is unique if it is not an inherent part of something else and is different from uniformity. A number is an inherent part of a smaller number either because it is needed for the smaller number to have meaning or because it can be expressed as a pattern of a single smaller number >1. The number 2 lacks uniqueness because it is an inherent part of creating the number 1, as evidenced by the existence of civilizations that had invented only 1 and 2 and by the absence of civilizations that invented only 1 but not 2. We need 2 to invent 1 or for 1 to have any meaning. We need both 1 and 2 in order to invent the concept of number. However, we do not need 3 to invent 1 and 2 as there are human groups that had invented 1 and 2 but not 3 [61].

All numbers are inherent in the number 1 as patterns of 1s but the property of uniqueness of the odd primes is not inherent in the pattern of 1s. Uniqueness is in contrast to uniformity and cannot exist independent of uniformity. While a prime can be expressed as a pattern of 1s, its uniqueness cannot. Every number (positive integer) can be uniquely defined by a pattern of 1s but this makes every number equally unique. Thus none is unique. The uniqueness of a number is based on the existence of numbers greater than 1 and the existence of non-unique numbers. Primes and non-primes are like odd and even or *yang* and *yin* and cannot exist independent of each other. The number 1 is unique since oneness is synonymous

with uniqueness. If 1 is unique, then 2 must be non-unique because it is an inherent part of creating the number 1.

The opposite of uniqueness is uniformity or not being able to be singled out. A prime also exists in a pattern, e.g., 18 is a pattern of the prime number 3. In such a pattern, the number 3 could not stand out as a unique individual. The uniformity property of a prime makes it possible for other subsequent primes to be uncovered as the unique. The number 23 is a prime because it is not a pattern of any other numbers greater than 1. The number 2 is essential for the number 1 to be unique and for other odd primes to be unique. For uniqueness to exist, the uniformity background must co-exist. Two is the first number of non-uniqueness and therefore has some uniqueness property and the related uniformity property. It is the most unique (the first number of non-uniqueness) and the most uniform (present in more patterns of 1s than any other non-unique numbers) among non-unique numbers.

If the building block of non-primes is the prime, it is only fair and logical to go down the hierarchy to ask what may be the building block of primes. The building block cannot be a number since prime number is the lowest level a number (positive integer) can be. If 1 is a prime, its building block must be 1 itself. The number 1 is also the building block of all other primes. A prime is a positive integer that can be built in only one way from its building block 1 by way of even number of sums of 1 but not of any other numbers greater than 1. How does a creative mind perceive the number 1? Of course, 1 represents uniqueness or oneness or a single smallest point of the whole. One is also uniformity or the single wholeness and is present everywhere or in every number or in every part of the whole. So, 1 embodies the ultimate duality of uniqueness and uniformity. To a creative mind, 1 and the duality are synonymous. The number equivalent of the duality concept is 1. Since 1 is the sole building block of primes, we can also say that the duality is the building block of primes. To use the duality as the building blocks of primes expresses the meaning of 1 as building blocks in a more fundamental way that is directly linked to the creation algorithm. The following shows that the creation algorithm can use the duality as building blocks to create primes. The mathematical model of the creation algorithm is the orderly creation of primes.

Postulate 1. The imagined domain All things created by the mind comes from imagination and the imagined world of the mind is termed the imagined domain. The content

of this domain consists of an infinite number of the basic building block of numbers, 1. There are infinite number of patterns of 1, each differ by its count of 1s. Each pattern, except that of a single 1, has the uniform property of having a count of 1s that is between two other patterns. The pattern of 2 is between the pattern of 1 and the pattern of 3. Since the contents of the imagined domain has no numbers smaller than 1, the pattern of a single 1 is not in between two other patterns and is therefore unique.

Postulate 2. The reality domain The reality domain is where the materialized creations of the mind exist. A prime is generated in the reality domain because of its uniqueness at the time of its creation. It subsequently exists in the reality domain because of its ability to initiate a pattern/uniformity. A prime is defined as a lawful creature of the mind that has the duality of uniqueness and uniformity. A non-prime is defined as a follower of a prime. The mind creates primes by following the two principles of the creation algorithm as postulated above: 1) to generate uniqueness by uniformity selection and 2) to maintain subsequent existence of the unique by uniqueness selection to form uniformity. Uniqueness selection is the process of species formation or forming follower numbers that share properties with the unique. For example, the follower numbers of 3 are 6, 9, 12, . . .3N, which share the uniform property of 3-ness and form the species of 3. A pattern of 1s or number moves from the imagined domain into the reality domain because it is either uniquely recognized by the mind or is necessary to maintain existence of the unique in the reality domain.

Creating primes. Prior to the creation of any numbers in the reality domain, the unique number in the imagined domain is 1. So the first goal is to generate 1 as the unique or prime in the reality domain. Since a prime must form a species or pattern in order to exist following its creation, the species of 1 is formed with 1 followed by the next closest number 2. In addition, to express uniqueness requires the simultaneous presence of uniformity. So, the species of 2 is formed to represent uniformity with 2 followed by the next closest number that shares the property of 2-ness, 4. Two is selected to represent uniformity because it is the only other pattern besides 1 that is available in the reality domain at this point when the species of 1 has not yet progressed beyond 2. Fig. 4A shows the contents of the reality domain at its time of creation. The prime/uniqueness/1/odd/*yang* and non-prime/uniformity/2/even/*yin* are

458 generated simultaneously and cannot exist independent of each other.

459 After the beginning stage of generating the reality domain, the mind is aware of both
460 the imagined domain and the reality domain. By comparing the two domains, the mind is
461 looking for the next prime or unique pattern among patterns in the imagined domain that have
462 no match in the reality domain. This pattern is now 3 and it is unique because it is the
463 smallest while all other patterns share the uniform property of having counts of 1s that are
464 between two patterns. To express 3 as a prime, the species of 3 (3, 6, 9) is formed in the
465 reality domain. To apply the new concept of 3-ness, all species are extended to the 3rd
466 position. The reality domain has now advanced from the beginning stage of 1 and 2 to the
467 next stage of 3-ness (Fig. 4B). At this stage, a number larger than 3 such as 4 expresses only
468 the concept already established such as 2-ness, 2 units of 2-ness. As soon as the stage of
469 3-ness has reached maximum completion, the mind is again ready to look for the next unique
470 pattern remaining in the imagined domain, which is now 5. From the concepts of 3-ness and
471 5-ness, the 4-ness of 4 is now recognized as the intermediate between 3 and 5. By applying
472 the concept of 5-ness and 4-ness, all number species are extended to the 5th position. The
473 species of 5 (5, 10, 15, 20, 25) is formed to express 5 as a prime. As soon as the concept of 5
474 has been applied to maximum completion, the mind is then ready to look for the next unique
475 pattern that remains in the imagined domain, which is 7 (Fig. 4C). In this way of iteratively
476 applying the same creation algorithm, an infinite number of primes can be generated. It is
477 easy to prove this. As reality is made of known primes and their composite numbers, one can
478 always find the next prime by merely finding the unique or smallest number remaining in the
479 imagined domain.

480 As the timing of creating each prime is determined by the time required to reach
481 maximum application of the concept of the previous prime, it is not randomly or arbitrarily
482 determined and hence would necessarily produce a pattern.

483 Because this creation algorithm of the mind can create primes, it is hereafter termed
484 the Prime Law. Since primes have the same property and meaning as creations of the mind,
485 the word ‘prime’ and the word ‘creation’ are interchangeable or synonymous. Therefore, the
486 ‘Prime’ Law also literally means the ‘Creation’ Law.

487

Creativity and the Prime Law

The uniformity selection principle suggests that the mind is capable of converting all that exist into a background upon which to base new imaginations. The uniqueness selection principle suggests that the mind is also programmed to adapt to existing paradigms. A human mind feels the need to fit in with the conventions of society but also feels the need to be unique or different from all other people. As Arthur Schopenhauer put it: "There is in the world only the choice between loneliness and vulgarity." Humans display polar opposite sides of creation-related character traits that are selected to coexist by the *yang* and *yin* principles. The uniformity selection principle values individualism, ambition, adventurism, self-centeredness, and distaste for routine labor, while the opposites are valued by the uniqueness selection principle. Both are essential for creation to go on, and all humans display unity of different degrees of both. Remarkably, experiments have shown that higher levels of right temporal alpha brainwaves enable people to come up with ideas which are further away from the obvious or well-known uses [62]. Such findings may also explain why right-brained or arts people are well known to be more creative [63]. Thus the Prime Law actually has neural basis and appears to be hardwired in the brain. The insatiable appetite for novelty of the human mind may come from it being hardwired with the uniformity selection principle, which thus may account for the constant creation of novelty in human civilizations.

The uniqueness selection principle also accounts for the inherent drive of humans to publicize their creative work once they have created something. If they do not work hard to present their creative work to the public and to have their work accepted and followed by others, their work would not count as a complete creation and would have no impact on the creative evolutionary process of humanity. The Prime Law suggests that this drive to have others to accept and follow one's own creative work may be essential to human creativity. Future experimental studies should test if this drive may also have neural basis.

The novel concept of uniformity selection seems obvious and explains the mind's insatiable appetite for novelty. The idea was inspired by death selection in nature or the inherent drive of life to stay away from death. Death is not disappearance of matter but is merely a return of matter from a unique high complexity ordered state (called life) to a uniformity state of less-ordered matter (called death). All life becomes the same in death in

terms of matter. Death is order-less uniformity state of life-building molecules. Each human mind has an inherent need to know what is the self or what is special or unique about the self. For the mind to stay away from uniformity/death, the mind needs to be unique. The only way to be unique is to be creative. But in order to be unique and creative, the mind needs to know or learn first what is the uniformity.

Uniformity selection describes the large step creations and uniqueness selection describes the small step progresses. Small step progresses are creations within a paradigm. Large step creations are changes in paradigm. It is widely noted that small step progresses cannot add up to large step creations [64]. The Darwinian theory is a creation law by a mind-less process. Mind is not needed in such a law for creation to occur. It is therefore hard to imagine that a mindless creation law can accurately describe the creation process of the mind. Nonetheless, it has been viewed as a Darwinian process of blind-variation and selective retention [54, 55]. While the notion of blind generation of ideas is debatable [56], this Darwinian view is actually a good description of the process of incremental advances within a paradigm.

To create (uniqueness) is more than simply making something new. To be new is necessary but not sufficient for a creation. A creation is not only new but is also unique among all potential new things that can be imagined by the mind and its uniqueness lies in the fact that it has the closest relationship to, or is the most smoothly adapted to, the whole pattern of what exist previously. This concept can be illustrated by the prime generation scheme as shown in Fig. 4C, which represents a time of existence that has no concept beyond 5-ness. Many numbers are missing in Fig. 4C and can qualify as new, such as 7, 11, 13, 14, 16, 17, 18, 19, 21, 22, 23, 24, and all numbers larger than 25. But only 7 is unique because it is the smallest missing number or because all numbers smaller than the unique number belong to what exist previously. A prime number or a creation is the one that is best adapted to the whole pattern of all that exist, whereas a non-prime number or something that is merely new is best adapted to a particular sub-pattern of the whole.

Why creation has the property of uniqueness? Because if it were not, the mind would not be able to recognize it from an infinity of choices. The mind does not create or select ideas by throwing a dice. If that were the case, the creations would not have the property of

uniqueness, and the mind would be unconscious of the properties of such creatures. Such creatures would lack any coherent logical relationship among them and would not be able to form the uniformity pattern to drive the next creation.

Indivisibility of primes

Uniqueness means that a number is not an inherent part of a smaller number greater than 1. A prime is not a pattern of any smaller number greater than 1, which means indivisible by any smaller number greater than 1. Indivisibility is therefore a secondary property of primes as the unique and should not in and of itself confer primality. The number 2 is indivisible but is not a prime because it lacks the uniqueness essence. It is an inherent part of creating the number 1 as the unique or prime.

The duality of unpredictability and regularity of primes

It is well known that primes seem to exhibit the duality of unpredictability and regularity. Such seemingly impossible unity of extreme opposites is what makes primes so interesting and mysterious [60]. However, the first fact of unpredictability remains unproven. It is the seeming randomness or unpredictability that makes the regularity of primes so striking and interesting. There exist a variety of formulas for either producing the *Nth* prime as a function of N or taking on only prime values. However, all such formulas require either extremely accurate knowledge of some unknown constant, or effectively require knowledge of the primes ahead of time in order to use the formula [65]. They do not really count as prediction. A true predictive formula should not make use of the knowledge of existing primes in order to predict the next future prime.

The newly discovered essence of primes can deduce the duality of unpredictability and regularity. When something can be predicted, it must belong to a pattern. As such, it is not unique and hence, by definition, not a prime. The essence of uniqueness rules out prediction of primes as a viable possibility. There is also another easy way to prove this. To predict primes means to predict uniqueness and in turn uniformity since uniqueness needs uniformity to have meaning. Uniformity is made of existing primes. So to predict primes is to predict *existing* primes, which is a logical non-sense. The uniformity essence of primes

demands that the formation of uniformity from a newly created prime or uniqueness must be regular and predictable. So the uniformity forming property of primes gives rise to the regularity of primes. Primes exist as regularly as possible in the uniformity. The creation of primes is fully determined by the orderly formation of uniformity by existing primes. The lawful rather than lawless way of creating primes explains why primes should follow some regularity patterns, such as the Prime Number Theorem. The unpredictability of individual primes explains why such a pattern cannot be completely precise or free of error margins.

Seeming randomness and real randomness

We use the phrase 'seeming randomness or deterministic randomness' to describe an outcome of a lawful process that is nonetheless unpredictable, like the creation of primes by the Prime Law. A population of such seemingly random outcomes should show a regularity pattern reflecting the lawfulness and regularity in the process leading to these outcomes. However, even the most precise pattern should still show some error margin reflecting the unpredictability or seeming randomness of the individual outcome. Primes have been found by many to show 'deterministic randomness' [66-70].

We define 'real randomness' as an unpredictable outcome of a lawless/arbitrary process like selecting a prime number from an infinity of numbers by playing a dice. Here the dice throw per se is not lawless/arbitrary/random. The lawless/arbitrary/random component in a lawless process involving the dice is connecting the dice arbitrarily with a meaningful concept or event that has no lawful connection to the dice, such as connecting prime numbers with the landing of a dice. For a process that involves both a lawful component (dice throw per se) and a lawless component (arbitrarily linking landing of dice with calling a number prime), the process is effectively lawless/arbitrary/random.

Both seeming-randomness and real-randomness are unpredictable but the error margins from a pattern are greater with real-randomness. A population of lawfully caused and predictable outcomes follows a precise pattern without any error margin. A population of lawfully caused but unpredictable outcomes follows a less precise pattern with some error margin like the square root of N. A population of lawlessly caused outcomes follows a rough pattern with huge error margins which could be so high as to render the pattern meaningless

or equivalent to no pattern. If whatever number that is selected from an infinity of numbers by playing a dice is defined as primes, we would obviously detect no meaningful patterns of primes in most cases, which is equivalent to saying that we could only have patterns with huge error margins. The error margin for a pattern of outcomes that are lawfully caused but unpredictable must necessarily be the smallest among patterns that cannot predict individual outcomes. Any smaller error margin would mean some degree of predictability. If we know that certain position of the tossing hand could cause a higher chance of landing heads while another position favoring tails, we could improve on the error margin but then the coin toss would not qualify as truly unpredictable. True unpredictability is shared by all kinds of randomness. Among these, the seeming-randomness or unpredictability of outcomes of a fully lawful process has the least amount of randomness or the smallest error margin from a regularity pattern.

There is a pattern that a fair coin toss follows, which says that the number of heads is equal to half of the number of toss N with an error of the square root of N. This pattern is a law that is valid based on logical reasoning alone. A fair coin toss must not have irregular or arbitrary/random bias toward the head or tail. Each landing of head or tail is fully determined by laws, such as the gravitational law, the exact position of the tossing hand, the wind, etc. A lawful process should produce reproducible outcomes. A coin toss is reproducible if the tossing conditions can be exactly reproduced. A coin toss is only seemingly random because of unpredictability. It is unpredictable because humans cannot measure all the physical parameters that determine the fall of a coin. Also, the laws are not biased to favor of either head or tail and remain unchanged timelessly. If a divine were to suddenly intervene for no reason to cause more landing of the head, the coin toss would be lawlessly caused and would display much wider error margin. If we only detect a seeming randomness in our coin toss with an error of the square root of N, we would be confident that everything is well and regular and no laws have been broken by either random accidents or deliberate intentions. But if we see a much wider variation than the square root of N, we would know that something is wrong or that some laws have been broken either accidentally or deliberately. The coin toss would be considered as unfair.

Since the RH means that the error margin from the pattern Li(N) is similar to the coin toss, one can prove the RH by showing that primes must have a regularity pattern which must have an error margin similar to the coin toss. This would be case if one can show that primes are lawfully caused and yet unpredictable. The creation process of prime is lawful and non-random or non-arbitrary. But the outcome of this process, i.e., calling a number a prime, is unpredictable. Because of the unpredictability, one simply cannot have a pattern of primes that is free of error margins. A pattern without error margins would mean predictability. However, because of the lawfulness of the process, the error margins must be the smallest possible among all kinds of unpredictable outcomes, which include those caused by either lawfully or lawlessly determined processes. Specifically, it must be smaller than the error margin of outcomes that involve a lawless process such as arbitrarily calling 6 a prime or calling the head of coin prime. The lawful creation of primes is similar to the coin toss in terms of being lawful and yet unpredictable. It is therefore expected that the two phenomena should have similar error margins. In both cases, the error margins are the smallest possible. Any bigger error margin would mean some degree of lawlessness in the process of creating primes or in the process of coin toss.

All other methods of generating or finding primes, such as division by smaller numbers and the sieve of Eratosthenes, are also orderly or deterministic. But unlike the Prime Law here, these methods cannot prove the unpredictability of primes. They define primes by the process of generating primes and thus do not give primes any meaning that is independent of the process: primes are whatever that are found by the process. Under the Prime Law, however, primes have the meaning of uniqueness/uniformity that is independent of the process of creating primes by the Prime Law. Uniqueness has meanings that are independent of the process of creating uniqueness. A creature has meanings that are independent of the process of creating the creature. Unpredictability is a property of the outcome and is independent of the process leading to the outcome. Both lawful and lawless processes can lead to unpredictable outcomes. That a coin falls half of the time head is an inherent property of the coin and is independent of the process of coin toss. If the property of the outcome is all defined or given by the process, then a lawful process simply cannot give the outcome the property of unpredictability or seeming randomness.

We have shown that primes must be unique and hence unpredictable because we can use the Prime Law to create primes by merely creating what is unique or unpredictable.

Evolution and the Prime Law

To understand evolution in nature, it may be a productive approach to first understand creative evolution in humans. One can then test if such an understanding may equally explain evolution in nature. If not, one at least would have succeeded in narrowing the field of possible solutions by excluding a major possibility. If intention can be excluded, then lawless chance creation theory, either the intention-less Darwinian theory or the arbitrary God theory of religions, may become valid by default as it would be the only alternative besides a law involving intentions. Remarkably, all known observations indicate that the Prime Law here derived by studying human creativity may well explain evolution in nature. There is a remarkable unity between the MGD theory and the Prime Law. Both involve linear and nonlinear processes. Both require maximum saturation to be reached in the linear process before an event in the nonlinear process can take place, which therefore can account for the nearly constant and yet discontinuous creations of novelty in relation to time. The first individual of a newly evolved novel species as a result of the nonlinear macro-evolutionary process could be viewed as a prime number, while the descendants or followers of the first individual as formed by the linear micro-evolutionary process could be viewed as composite numbers.

To further establish the role of the Prime Law in evolution, one could aim to demonstrate or strengthen the following. 1. All fundamentally novel species as a result of the nonlinear macro-evolutionary process were unique at the time when the first individuals of the species first appeared. There were no repeated creations of the same kind of species as such repeats would be copies rather than unique. 2. The wide and persistent existence of certain abstract and gratuitous patterns or beauties in nature that have no apparent functional relevance, such as the Golden Ratio or 5 toes rather than 4 or 6 (pentadactyl pattern), may be because they are the most unique. The Prime Law offers a viable angle to understand this mystery while the chance theories of Darwin and Kimura are completely clueless [71]. 3. There may be only one unique universe and the fine tuning or just right property of our

universe may be a result of uniformity selection. Just right is unique. It has long been noted by physicists that the values of over a dozen fundamental physical constants of the universe are precisely fined tuned for life to exist [72]. If the values are slightly different, life could not exist [73]. 4. Most things in nature are at stable maximum or optimum (Pareto optimum) saturation balance. 5. The Principle of Least Action is a most fundamental and unifying physical law of nature and may be embedded in the notion of the Prime Law that only the unique (and its followers) exists in nature. The minimum is unique. 6. Nature is comprehensible, as Einstein put it famously: "The most incomprehensible thing about the universe is that it is comprehensible." This easily follows if the same law underlies both creative evolutions in nature and human creativity. 7. Nature is written in the language of mathematics. It thus follows that the foundation of mathematics, the primes and the Prime Law, should also be the foundation for the universe. 8. Only intention rather than chance can cause significant reduction in randomness (in the genomes) as found during complexity increases in macro-evolution (Fig. 1A). Intention or mind is inherent in the Prime Law. A chance creation theory or an omnipotent God theory capable of any arbitrary or unlawful actions as described by the major religions would mean the exact opposites of all of the above.

As primes are infinite, the Prime Law means that the creation process would be endless. If human creativity and evolution in nature use different laws, one intentional and the other chance or arbitrary, then human creative evolutionary process could be prematurely stopped by chance. Thus, for human creative process to be endless and thus meaningful, it is essential that the same Prime Law is also the foundation for evolution in nature.

Summary:

Evolution, human creations, and prime numbers share a common feature of being both seemingly random (unpredictable) and orderly. They all also involve both linear and nonlinear processes, and show a similar pattern of nearly constant and yet seemingly random creation of novelty in relation to time. Such characteristics are inconsistent with chance or arbitrary creations and can be explained by a creation algorithm that nonetheless cannot be predictive. This algorithm appears to be hardwired in the human brain.

Acknowledgements:

Supported by the National Natural Science Foundation of China grant 81171880 and the National Basic Research Program of China grant 2011CB51001.

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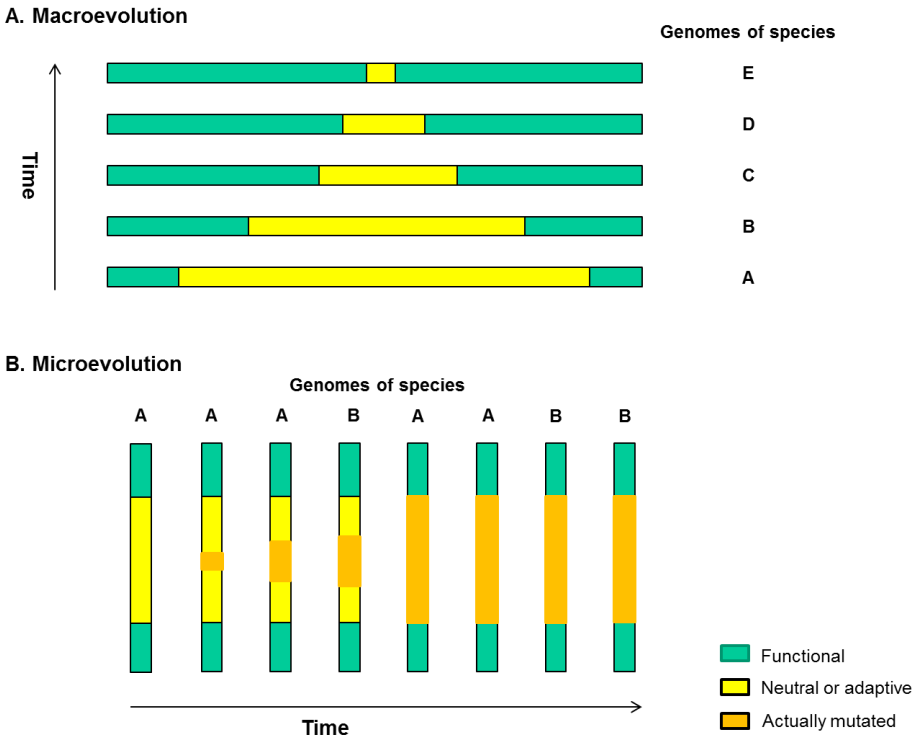
Figure legends:

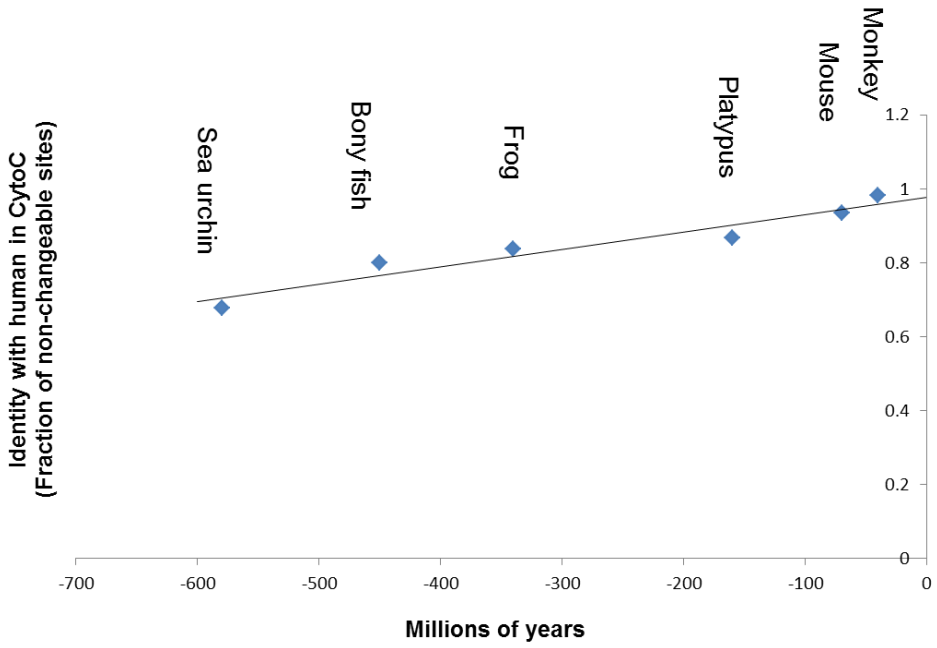
Figure 1. Model of evolution by the MGD theory. Schematic representation of macroevolution (A) and microevolution (B). Yellow color represents allowed or tolerable mutant sites in a sequence. Orange color represents sites where actual mutations have occurred. Macroevolution involves increases in complexity and decreases in the fraction of tolerable mutant sites. Microevolution involves no changes in complexity and in the fraction of tolerable mutant sites.



888 **Figure 2. The nearly constant rate of complexity increases.** The fraction of identical
889 residues between human and a lower complexity species is equivalent to the fraction of
890 non-changeable sites in the lower complexity species. The fraction of identical residues in
891 cytochrome C (identity divided by length) between human and each of the species listed in
892 the figure is plotted against the separation time between human and each of the listed species.
893 Data for plots were obtained using homo cytochrome C to BLASTP the protein database of
894 Genbank.

895

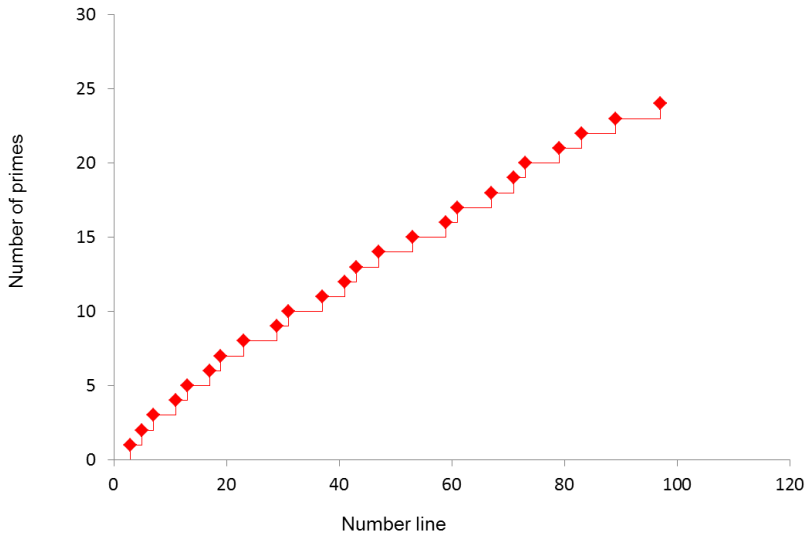
896



897

Figure 3. Staircases of prime numbers. The graph counts the cumulative number of primes up to 100 (A), and 10000 (B).

A



B

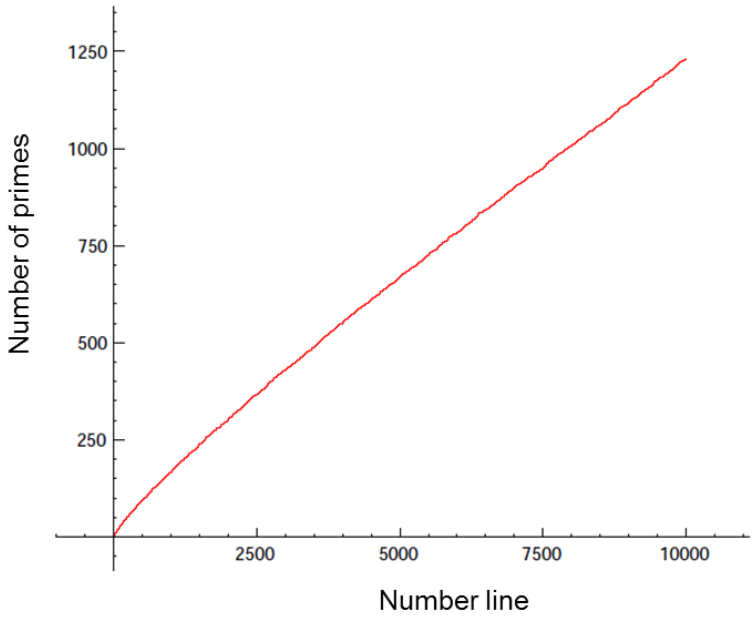


Figure 4. Creating primes by the iterations of the creation algorithm. A. The contents of the reality domain at the time of creation. The species of 2 is listed not because 2 is a prime but because it is an inseparable part of creating the first prime 1. **B.** The contents of the reality domain at the time of 3. **C.** Subsequent progression of the reality domain. From left to right represents the number species with each number increasing in value from the previous number by the unit value of the beginning number; the species terminates at the Pth position where P is the numeric value of the last known prime (P>2). Successive prime numbers from small to large are listed on the left side column in the order from top to bottom. The table can be expanded in a prime by prime manner over time to infinity, in both the vertical direction from top to bottom and the lateral direction from left to right.

A

1	2
2	4

B

1	2	3
2	4	6
3	6	9

C

1	2	3	4	5	…	1N
2	4	6	8	10	…	2N
3	6	9	12	15	…	3N
5	10	15	20	25	…	5N
…	…	…	…	…	…	…
P	P2	P3	P4	P5	…	PN