

Advanced Lectures in Mathematics  
Volume XII

# Cohomology of Groups and Algebraic $K$ -theory

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# Preface

Cohomology of groups is a fundamental tool in many subjects in modern mathematics. One important generalized cohomology theory is the algebraic  $K$ -theory, and algebraic  $K$ -groups of rings such as rings of integers and group rings are important invariants of the rings. They have played important roles in algebra, geometric and algebraic topology, number theory, representation theory etc. Cohomology of groups and algebraic  $K$ -groups are also closely related. For example, algebraic  $K$ -groups of rings of integers in number fields can be effectively studied by using cohomology of arithmetic groups.

To introduce these fundamental concepts to graduate students and non-experts, an international summer school titled “Cohomology of groups and algebraic  $K$ -theory” was organized and held at the Center of Mathematical Sciences (CMS), Zhejiang University, from July 1 to July 13, 2007. Besides lecture series by several distinguished speakers on algebraic  $K$ -theory, algebraic  $K$ -groups of rings of integers in number fields and group rings which are fundamental in geometric topology, there were also some research talks.

This volume consists of expanded lecture notes of some talks at the summer school, two fundamental papers of D.Quillen<sup>1</sup>, a set of lecture notes of M.Karoubi<sup>2</sup> and several other papers on related topics. Together, they present a coherent picture of the topics under discussion. We hope and expect that this book will be a very valuable reference on and introduction to these topics for both graduate students and mathematicians who want to learn some basics in cohomology of groups and algebraic  $K$ -groups, and related recent developments.

All the papers in this proceedings are carefully refereed. We would like to thank the referees for their kind help. We would also like to thank the staff members of CMS, Zhejiang University, for their help during the summer school

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<sup>1</sup>D.Quillen, (1) *Higher algebraic K-theory. I*, in *Algebraic K-theory, I: Higher K-theories* (Proc. Conf., Battelle Memorial Inst., Seattle, Wash., 1972), pp. 85–147. Lecture Notes in Math., Vol. 341, Springer, 1973, and (2) *Finite generation of the groups  $K_i$  of rings of algebraic integers*, in *Algebraic K-theory, I: Higher K-theories* (Proc. Conf., Battelle Memorial Inst., Seattle, Wash., 1972), pp. 179–198. Lecture Notes in Math., Vol. 341, Springer, 1973.

<sup>2</sup>M.Karoubi, *Lectures on K-theory*, in *Contemporary developments in algebraic K-theory*, pp. 1–95 (electronic), ICTP Lect. Notes, XV, Abdus Salam Int. Cent. Theoret. Phys., Trieste, 2004.

and the preparation of this proceedings. Finally, we would like to thank Springer for their permission for us to reprint the two papers of D.Quillen and ICTP, Trieste, for their permission to reprint the paper of M.Karoubi mentioned above.

Lizhen Ji (the Managing Editor)

Kefeng Liu

Shing-Tung Yau

November 2008

# Contents

<i>Arthur Bartels and Wolfgang Lück: On Crossed Product Rings with Twisted Involutions, Their Module Categories and <math>L</math>-Theory</i> .....	1
<i>Oliver Baues: Deformation Spaces for Affine Crystallographic Groups</i> .....	55
<i>Kenneth S. Brown: Lectures on the Cohomology of Groups</i> .....	131
<i>Daniel R. Grayson: A Brief Introduction to Algebraic <math>K</math>-Theory</i> .....	167
<i>Daniel Juan-Pineda and Silvia Millan-López: The Braid Groups of <math>\mathbb{R}P^2</math> Satisfy the Fibered Isomorphism Conjecture</i> .....	187
<i>Max Karoubi: <math>K</math>-Theory, an Elementary Introduction</i> .....	197
<i>Max Karoubi: Lectures on <math>K</math>-Theory</i> .....	217
<i>Wolfgang Lück: On the Farrell-Jones and Related Conjectures</i> .....	269
<i>Stratos Prassidis: Introduction to Controlled Topology and Its Applications</i> .....	343
<i>Hourong Qin: Lecture Notes on <math>K</math>-Theory</i> .....	387
<i>Daniel Quillen: Higher Algebraic <math>K</math>-Theory: I</i> .....	413
<i>Daniel Quillen: Finite Generation of the Groups <math>K_i</math> of Rings of Algebraic Integers</i> .....	479
<i>David Rosenthal: A User's Guide to Continuously Controlled Algebra</i> .....	489
<i>Christophe Soulé(Notes by Marco Varisco): Higher <math>K</math>-Theory of Algebraic Integers and the Cohomology of Arithmetic Groups</i> .....	503