High-speed three-dimensional shape measurement system using a modified two-plus-one phase-shifting algorithm

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Abstract. This paper describes a high-resolution, real-time, three-dimensional shape measurement system using the modified two-plus-one phase-shifting algorithm. The data acquisition speed is as high as 60 frames/s with an image resolution of 640×480 pixels per frame. Experiments demonstrated that the system was able to acquire the dynamic changing objects such as facial geometric shape changes when the subject is speaking, and the modified two-plus-one phase-shifting algorithm can further alleviate the error due to motion. Applications of this system include manufacturing, online inspection, medical imaging, compute vision, and computer graphics. [DOI: 10.1117/1.2802546]

Subject terms: 3-D shape measurement; phase-shifting; real-time; high-resolution.

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1 Introduction

With the development of the hardware technologies including the computer, camera, and digital display, real-time 3-D shape measurement is becoming practical and increasingly important. The applications are diverse, including manufacturing, medical research and clinical practices, compute vision, and computer graphics.

In general, for real-time 3-D shape measurement using a structured-light-based method, the fewer the number of fringe images used, the faster the speed achieved. High-speed 3-D shape measurement systems based on single color images have been developed.[1–4] However, the problem of using color is that the measurement is affected by the color of the object. Algorithms based on single black-and-white (B/W) fringe image[5] or double B/W fringe images[6,7] can also achieve faster data acquisition speed than an algorithm based on three or more fringe images. However, since the phase cannot be resolved directly from the single or dual fringe images, the measurement, in general, has strict requirements for the measurement environment or the complexity of the geometric shape of the objects. In contrast, a three-step phase-shifting algorithm is able to solve the phase uniquely from three fringe images. Therefore, for high-speed, accurate measurement of complex geometric shapes, a three-step phase-shifting algorithm is the first choice.

In recent years, we have endeavored to develop a high-resolution, real-time 3-D shape measurement system.[8–10] By implementing a fast three-step phase-shifting algorithm,[11] Zhang and Huang developed a high-resolution, real-time 3-D shape measurement system that is able to acquire, reconstruct, and display the relative 3-D shapes at a speed up to 40 frames per second (fps), with an image resolution of 532×500 pixels per frame, all in an ordinary computer.[12] Recently, we developed a 3-D absolute coordinates measurement system by encoding a marker in the projected fringe images and detecting it in real-time.[13] These systems were able to measure slow motion like normal facial expressions satisfactorily. Our previous work demonstrated the potentials of high-resolution, ream-time 3-D shape measurement in various applications such as medical imaging, feature films, movies, games, etc. However, all our previous systems encountered the problem of fast motion, such as the facial geometric changes during speaking. We were frequently requested to develop a system that is capable of measuring the facial geometric shape changes when the human subject is speaking at a normal speed. This work was motivated by such requests. Based on our previous systems developed, we developed a new system to meet most of their requirements. This paper presents a system that is able to acquire the 3-D geometric shapes at 60 fps with a resolution of 640×480 pixels per frame. Instead of using a three-step phase-shifting algorithm, a modified 2+1 phase-shifting algorithm was employed to further alleviate the measurement errors due to motion.

To reduce the measurement errors due to vibration, Angel and Wizinowich proposed a 2+1 phase-shifting algorithm.[14,15] In this approach, two fringe images having 90° phase shift are rapidly captured and a third flat image is collected that is the average of two fringe images with a phase shift of 180°. This algorithm has found limited use because the small number of data frames in this algorithm makes it susceptible to errors resulting from phase-shifter nonlinearity and calibration.[16] In this research, we are trying to measure dynamically changing objects. The error caused by motion is similar to that caused by vibration, i.e., the error occurs when the faster geometric shape changes are presented.

Unlike the laser interferometry based system, our system uses a digital video projector to create the fringe patterns, and the errors resulting from phase shift are not there due to its digital fringe generation nature. Unlike the 2+1 phase-shifting algorithm proposed by Angel–Wizinowich, we capture the third flat image directly by projecting a computer generated uniform flat image instead of averaging two phase-shifted fringe images. Hence, only three images instead of four images are used for our algorithm, therefore we call it a modified 2+1 phase-shifting algorithm. By implementing this algorithm into our real-time 3-D shape measurement system where a three-step phase-shifting algorithm is used, we maintain the measurement speed while reducing the errors caused by motion using a three-step phase-shifting algorithm. Because the phase information is encoded in all three images for a three-step phase-shifting algorithm and only two images for a 2+1 phase-shifting algorithm, and a flat image is less sensitive to the motion blur than an image with fringe stripes, a 2+1 phase-shifting algorithm can alleviate the error due to motion of a three-step phase-shifting algorithm. Our experiments demonstrated that this system can satisfactorily measure the dynamically facial geometries when the subject is speaking at even faster than normal speed.

Section 2 describes the principles of the system. Section 3 shows the experimental results, and Section 4 concludes the paper.

2 Principle

Phase-shifting methods for 3-D shape measurement have long been employed in optical metrology for their speed and accuracy. For these methods, a number of (normally $\geqslant 3$) fringe images are recorded and the phase is computed from the fringe images. 3-D coordinates can be obtained based on triangulation. Various phase-shifting methods have been developed, such as three-step, four-step, and five-step algorithms.[17] In general, the more fringe image used, the higher the accuracy obtained, and the slower the measurement speed achieved. For real-time, accurate, 3-D shape measurement, a three-step phase-shifting algorithm is a good choice. The three-step phase-shifting algorithm with a phase shift of 120° has the advantage of its symmetry; however, the measurement error is sensitive to any fringe image errors caused by various sources such as motion blur. To alleviate this problem, in this research we replace the third image by a uniform flat image and the first two images with a phase shift of 90°, which is called the modified 2+1 phase shifting algorithm.

2.1 *Three-Step Phase Shifting Algorithm*

A three-step phase shifting algorithm with a phase shift of 120° can be written as

$$I_1 = I'(x,y) + I''(x,y)\cos(\phi(x,y) - 120^\circ), \tag{1}$$

$$I_2 = I'(x,y) + I''(x,y)\cos(\phi(x,y)), \tag{2}$$

$$I_3 = I'(x,y) + I''(x,y)\cos(\phi(x,y) + 120^\circ), \tag{3}$$

where $I'(x,y)$ represents the average intensity; $I''(x,y)$ the intensity modulation; and $\phi(x,y)$ the phase to be resolved. Solving these equations simultaneously, we can obtain the phase

$$\phi(x,y) = \tan^{-1}\frac{\sqrt{3}(I_1 - I_3)}{(2I_2 - I_1 - I_3)} \tag{4}$$

and 2-D flat texture image

$$I'(x,y) = (I_1 + I_2 + I_3)/3. \tag{5}$$

2.2 2+1 *Phase-Shifting Algorithm*

Our research focus is to develop a real-time 3-D shape measurement system that is able to measure dynamically changing objects, such as facial expressions. To alleviate the measurement errors caused by motion, we replace the third image by a uniform flat image and choose a phase-shift of 90°, which is called the modified 2+1 phase shifting algorithm. The intensity of the fringe image therefore becomes

$$I_1 = I'(x,y) + I''(x,y)\sin(\phi(x,y)), \tag{6}$$

$$I_2 = I'(x,y) + I''(x,y)\cos(\phi(x,y)), \tag{7}$$

$$I_3 = I'(x,y). \tag{8}$$

Solving Eqs. (6)–(8), we have

$$\phi(x,y) = \tan^{-1}\frac{I_1 - I_3}{I_2 - I_3}, \tag{9}$$

where phase $\phi(x,y)$ in Eq. (4) or Eq. (9) is the so-called modulo 2π at each pixel with value ranging from $[-\pi, +\pi)$. A phase unwrapping algorithm is necessary to remove the 2π discontinuities if multiple fringe stripes are used.[18] Once the continuous phase map is obtained, the phase at each pixel can be converted to xyz coordinates of the corresponding point on the object surface through calibration.[19]

To realize automatic 3-D measurement, the flat image has to be determined for a 2+1 phase shifting algorithm. In this research, the fringe images are identified by analyzing the three fringe images automatically. In order to detect the three fringe images, the flat fringe image has to be identified first. We define a function

$$\Delta_i = \sum_x \left|\frac{\partial I_i(x,y)}{\partial x}\right| + \sum_y \left|\frac{\partial I_i(x,y)}{\partial y}\right| \tag{10}$$

to find the flat image. The fringe image with the minimum value is the flat image since this image has less intensity variations than the other two fringe images. Because three images are projected and captured sequentially and repeatedly, the other two images are uniquely determined once the flat image is known.

The modified 2+1 phase-shifting algorithm has the following advantages over three-step phase-shifting algorithm:

- *Faster motion measurement*. The phase is only encoded with two fringe images and the third image is less sensitive to motion than the image with fringe stripes. Therefore, the measurement error due to motion is less for the modified 2+1 phase-shifting algorithm.

- *Direct high quality texture acquisition.* For the modified 2+1 phase shifting algorithm, I_3 is actually the flat image that can be used for texture mapping purposes. Although it is not very important for optical metrology, the high-quality texture along with the aligned 3-D geometry is highly important for a lot of applications such as face recognition, computer vision, computer graphics, etc. The texture images can also be obtained from a three-step phase-shifting algorithm. However, it requires that the three fringe images have to have the exact phase shift and the profile of the fringe has to be ideally sinusoidal, otherwise, the texture image quality is not sufficiently good.
- *Faster processing speed.* Since the phase can be obtained from a direct intensity ratio computation rather than an arctangent function computation, the computation speed is much faster. We found that processing speed of the modified 2+1 phase-shifting algorithm is more than two times faster than that of the three-step phase-shifting algorithm.

2.3 *System Calibration*

In this research, we used the system calibration method discussed in Ref. 19. The core of this calibration technique is to enable the projector to "capture" the images like a camera, so that the calibration of a structured light system becomes a well studied stereo system. Once the system is calibrated, two transformation matrices are obtained, from the world coordinates to image coordinates for the camera and the projector. Through the absolute phase, absolute xyz coordinates can be computed pixel by pixel.

If only the linear model is considered, the camera parameter matrix A_c and projector matrix A_p can be obtained after calibration. They are as follows:

$$A_c = \begin{bmatrix} a^c_{11}, & a^c_{12}, & a^c_{13}, & a^c_{14} \\ a^c_{21}, & a^c_{22}, & a^c_{23}, & a^c_{24} \\ a^c_{31}, & a^c_{32}, & a^c_{33}, & a^c_{34} \end{bmatrix}, \tag{11}$$

$$A_p = \begin{bmatrix} a^p_{11}, & a^p_{12}, & a^p_{13}, & a^p_{14} \\ a^p_{21}, & a^p_{22}, & a^p_{23}, & a^p_{24} \\ a^p_{31}, & a^p_{32}, & a^p_{33}, & a^p_{34} \end{bmatrix}. \tag{12}$$

These coordinate transformation matrices include the translation and rotation matrix from the camera or projector coordinates to the world coordinates. One unique fixed world coordinate relative to the projector and camera can be selected based on one calibration image pair. The relationships between the world coordinates (x,y,z) (or the absolute coordinates) and the camera pixel coordinates (u^c,v^c) and the projector pixel coordinates (u^p,v^p) are

$$s^c[u^c,\ v^c,\ 1]^T = A^c[x,\ y,\ z,\ 1]^T, \tag{13}$$

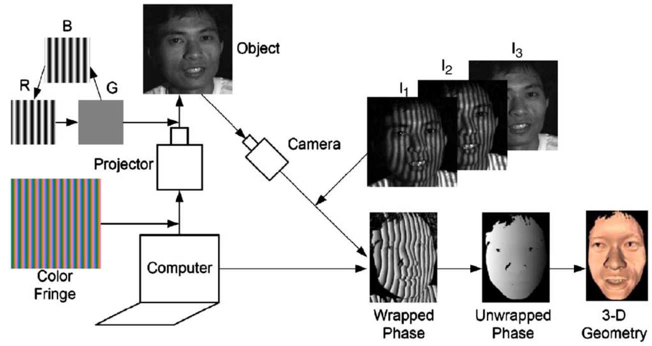

Fig. 1 Schematic diagram of the shape measurement system.

$$s^p[u^p,\ v^p,\ 1]^T = A^p[x,\ y,\ z,\ 1]^T, \tag{14}$$

where s^c, s^p are camera and projector scaling factors, respectively.

Once the absolute phase ϕ_a is obtained, the relationship between the camera coordinates and the projector coordinates can be established

$$\phi^c_a(u^c,v^c) = \phi^p_a(u^p) = \phi_a. \tag{15}$$

From Eqs. (11)–(15), the absolute coordinates can be resolved

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a^c_{11} - u^c a^c_{31} & a^c_{12} - u^c a^c_{32} & a^c_{13} - u^c a^c_{33} \\ a^c_{21} - v^c a^c_{31} & a^c_{22} - v^c a^c_{32} & a^c_{23} - v^c a^c_{33} \\ a^p_{11} - u^p a^p_{31} & a^p_{12} - u^p a^p_{32} & a^p_{13} - u^p a^p_{33} \end{bmatrix}^{-1} \begin{bmatrix} u^c a^c_{34} - a^c_{14} \\ v^c a^c_{34} - a^c_{24} \\ u^p a^p_{34} - a^p_{14} \end{bmatrix}. \tag{16}$$

Therefore, once the absolute phase is known and the system is calibrated, the 3-D coordinates can be obtained.

2.4 *System Setup*

Figure 1 shows the layout of the system. A computer generates the color fringe pattern that is encoded the three fringe images into its three primary color channels red, green, blue. This color fringe pattern is sent to a single-chip DLP projector (PLUS U5-632h) and projected in B/W mode sequentially and repeatedly at a frame rate of 120 fps. A high-speed charge-coupled device camera (Pulnix TM-6740CL) synchronized with the projector is used to capture the reflected fringe images at a speed of 180 fps. Based on the modified 2+1 phase-shifting algorithm used, any successive three fringe images can be used to reconstruct one 3-D geometry through phase wrapping, phase unwrapping, and system calibration procedures. Therefore, the 3-D shape measurement speed is 60 fps. The third image, I_3, is a flat image that can be used as texture for texture mapping or other purposes.

Figure 2 shows the timing chart of this 3-D shape measurement system. Due to the speed limit of the B/W camera used (maximum speed of 200 fps), three fringe images need to be taken in two projection cycles. Since the red and blue color channels are captured successively, we encode

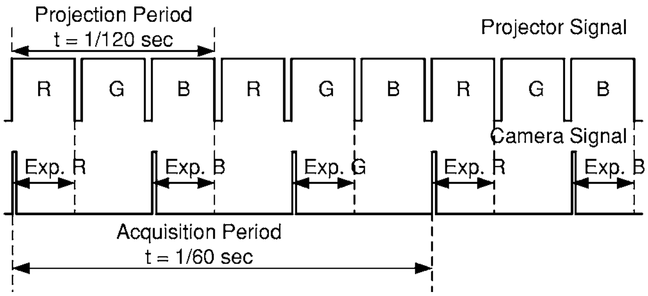

Fig. 2 Timing chart of the measurement system.

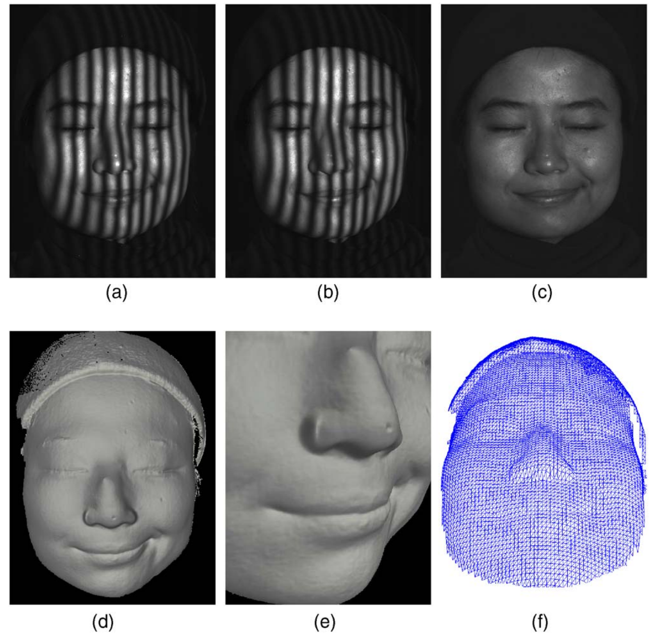

Fig. 4 Measurement results of facial geometry using the modified 2+1 phase-shifting algorithm introduced (a) I_1; (b) I_2; (c) I_3; (d) the reconstructed 3-D geometry in shaded mode; (e) zoom-in view of 3-D geometry; and (f) 3-D geometry in wireframe mode.

the color fringe pattern with red being I_1, blue being I_2, and green being I_3. Because I_1 and I_2 are captured rapidly and the flat image I_3 is less sensitive to motion, this coding algorithm can alleviate the measurement errors due to motion. Figure 3 shows a photograph of the hardware system developed.

The Pulnix TM-6740CL camera is a 10-bit camera; however, we found that acquiring the 10-bit data at 180 fps by an ordinary computer is difficult. Therefore, all data presented in this paper are acquired at 8 bit. Even though the data transferring rate was reduced by using the 8-bit format, our experiment showed that the double-buffering technique used in our previous system is not enough and some frames are lost.[12] To resolve this problem, a pentuple-buffering data acquisition technique was employed. A pentuple-buffering technique is similar to the double-buffering technique: when the frame grabber is grabbing the ith frame, the computer is copying the $(5-i)$th frame to the computer memory. Once the whole images have been acquired, they are saved to hard drive for postprocessing.

3 Experiment

To verify the performance of the real-time 3-D shape measurement system, we measured a human face using the modified 2+1 phase-shifting algorithm, as shown in Fig. 4. This figure shows that the 3-D shape can be well acquired using this algorithm. It should be noted that all the data presented in this paper were smoothed by a 5×5 Gaussian filter to reduce the most significant random noise.

To test the capability of the system, we measured a human face when the subject was speaking at even faster than normal speed. Figure 5 shows a photograph captured during the experiment. The left side of the image shows the subject while the right side shows the real-time reconstructed geometry. This new system can achieve 60 fps measurement speed, therefore it can measure fast motion. Figure 6 shows several frames of a speaking face. The first row shows the full face, while the second row shows the zoom-in view of the mouth region, where the fastest motion occurs. It can be seen that the profiles of the mouth are very smooth. This means that the 3-D shapes are well captured when the subject is speaking. This example demonstrates

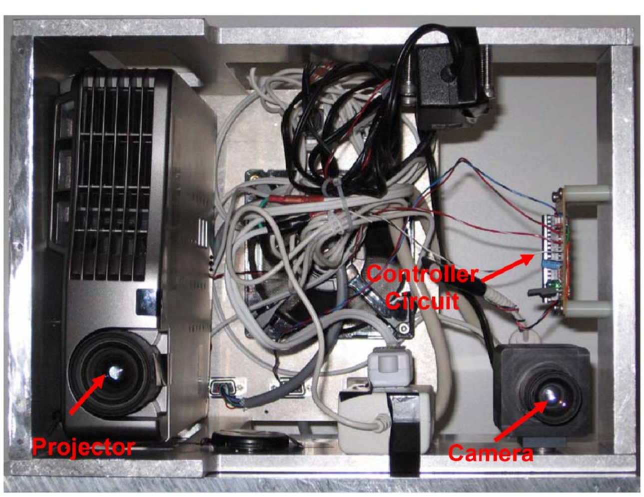

Fig. 3 Photograph of the measurement system.

Fig. 5 Geometric video acquired at 60 fps with our system. The left figure is the real-time reconstructed 3-D result. The right figure is the subject.

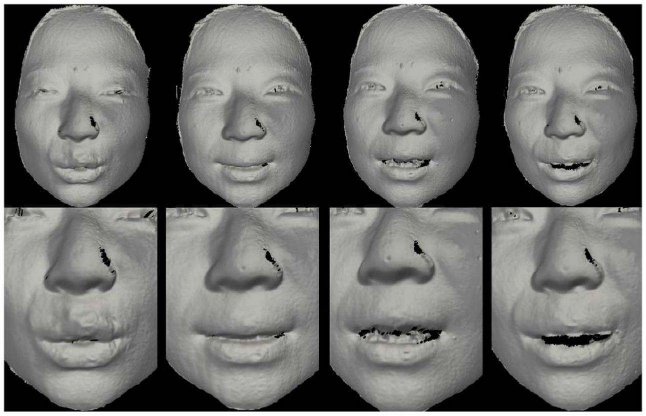

Fig. 6 The speaking face. The first row shows some typical frames of the facial geometries when the subject is speaking. The second row shows corresponding zoom-in view of the above 3-D geometries.

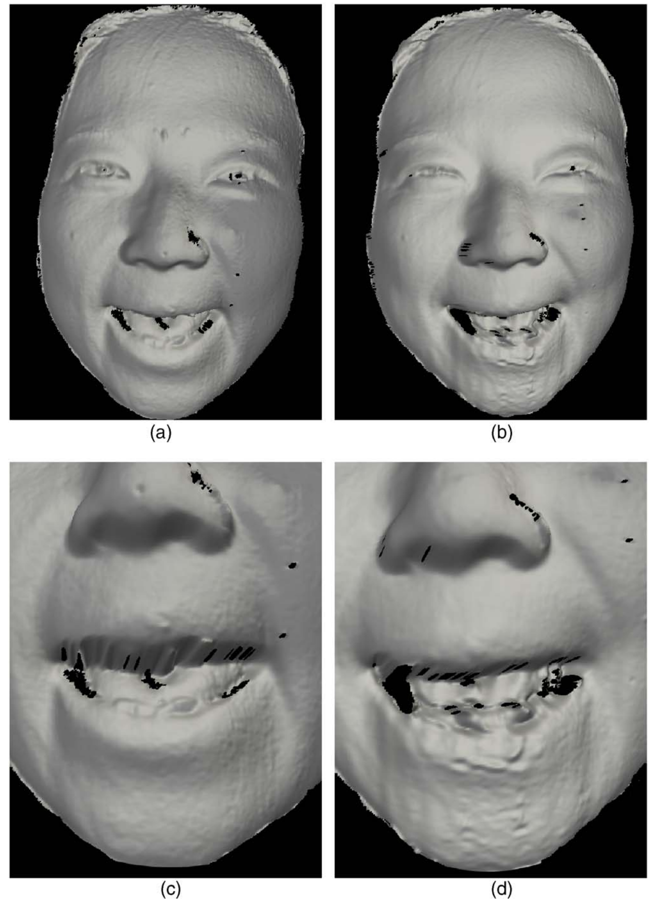

Fig. 7 Comparison between the measurement results using 2+1 phase-shifting algorithm and three-step phase-shifting algorithm: (a) 3-D geometry using 2+1 phase-shifting algorithm; (b) 3-D geometry using three-step phase-shifting algorithm; (c) zoom-in view of mouth area for 3-D geometry in (a); and (d) zoom-in view of mouth area for 3-D geometry in (b).

that our system can capture the geometric shapes accurately when the human subject is speaking at normal speed.

As a comparison, we measured human facial expressions when speaking using both algorithms. Figure 7 shows the measurement results. During the experiment, the subject was asked to speak the same sentences repeated at the same speed so that the constant facial motions could be generated at the same speed. The first row shows the 3-D geometries using the 2+1 phase-shifting algorithm and the three-step phase-shifting algorithm. The second row shows the corresponding zoom-in view of the area close to the mouth. The measurement result using the 2+1 phase-shifting algorithm is smooth while that using the three-step phase-shifting algorithm has errors near the mouth region. It can also be seen that the overall noise of the 3-D geometry produced by the 2+1 phase-shifting algorithm is greater than that produced by the three-step phase-shifting algorithm.

Figure 8 shows the cross section of the 3-D profile of the measurement results using both algorithms. The top row shows the measurement result using the 2+1 phase-shifting algorithm, while the second row shows the measurement result using the three-step phase-shifting algorithm. This figure clearly shows that near the mouth region where the faster motion occurs, the 2+1 phase-shifting algorithm produces better results than three-step phase-shifting algorithm.

Our experiments show that although the overall noise is greater, the 2+1 phase-shifting algorithm yields better measurement results near the mouth region where the motion is faster. Since the errors of the geometry near the mouth region obtained using the 2+1 phase-shifting algorithm are smaller than those obtained using the three-step phase-shifting algorithm, these experiments demonstrate that the system can obtain satisfactory results using the 2+1 phase-shifting algorithm.

4 Conclusion

This paper has described a 60 fps 3-D shape measurement system using the modified 2+1 phase-shifting algorithm. By replacing the three-step phase-shifting algorithm of our previous system with this new algorithm, the measurement errors caused by motion were reduced. This 60 fps system can acquire the facial geometric shape changes when the subject was speaking even at a faster than normal speed. The hardware system was developed and tested. Experimental results were presented to demonstrate the potential capability of the system. This new system can acquire significantly better data for dynamically changing objects. The technology presented in this paper will benefit many applications such as medical imaging, gaming, animation, computer vision, computer graphics, etc.

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References

1. K. G. Harding, "Phase grating use for slop discrimination in moiré contouring," *Proc. SPIE* 265–270 (1991).
2. Z. J. Geng, "Rainbow 3-D camera: New concept of high-speed three vision system," *Opt. Eng.* **35**, 376–383 (1996).
3. C. Wust and D. W. Capson, "Surface profile measurement using color fringe projection," *Mach. Vision Appl.* **4**, 193–203 (1991).
4. P. S. Huang, Q. Hu, F. Jin, and F. P. Chiang, "Color-encoded digital

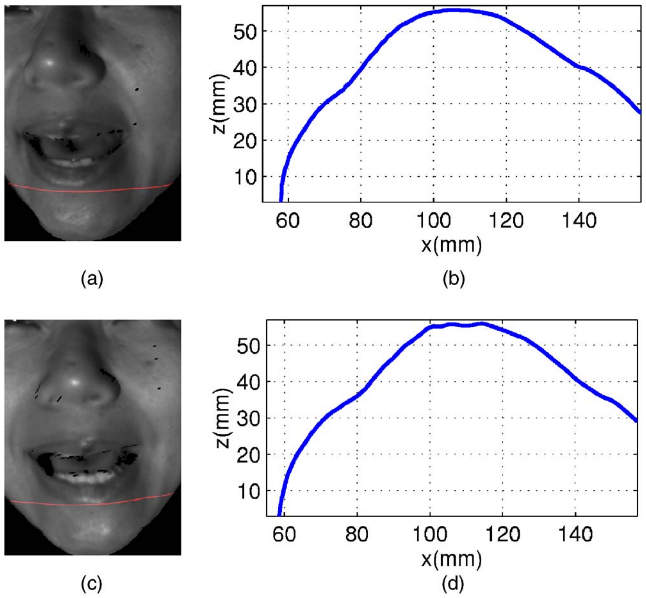

Fig. 8 Cross section of the 3-D profile near the mouth area: (a) cross section line of the 3-D result by using the 2+1 phase-shifting algorithm; (b) (x,z) coordinates plot of the line shown in (a); (c) cross section line of the 3-D result by using the three-step phase-shifting algorithm; and (d) (x,z) coordinates plot of the line shown in (c).

fringe projection technique for high-speed three-dimensional surface contouring," *Opt. Eng.* **38**, 1065–1071 (1999).
5. M. Takeda and K. Mutoh, "Fourier transform profilometry for the automatic measurement of 3-D object shape," *Appl. Opt.* **22**, 3977–3982 (1983).
6. S. Almazán-Cuéllar and D. Malacara-Hernández, "Two-step phase-shifting algorithm," *Opt. Eng.* **42**, 3524–3531 (2003).
7. C. Quan, C. J. Tay, X. Kang, X. Y. He, and H. M. Shang, "Shape measurement by use of liquid-crystal display fringe projection with two-step phase-shifting," *Appl. Opt.* **42**, 2329–2335 (2003).
8. S. Zhang, "High-speed 3d measurement based on a digital fringe projection technique," Master's thesis, Stony Brook University, State University of New York (2003).
9. S. Zhang and P. Huang, "High-resolution, real-time 3-D shape acquisition," in *Proc. IEEE Comp. Vis. and Patt. Recogn. Workshop*, Washington, DC, Vol. 3, pp. 28–37 (2004).
10. S. Zhang, "High-resolution, real-time 3-D shape measurement," PhD thesis, Stony Brook University, State University of New York (2005).
11. P. S. Huang and S. Zhang, "Fast three-step phase shifting algorithm," *Appl. Opt.* **45**, 5086–5091 (2006).
12. S. Zhang and P. S. Huang, "High-resolution, real-time three-dimensional shape measurement," *Opt. Eng.* **45**, 123601 (2006).
13. S. Zhang and S.-T. Yau, "High-resolution, real-time 3D absolute coordinate measurement based on a phase-shifting method," *Opt. Express* **45**, 2644–2649 (2006).
14. J. R. P. Angel and P. L. Wizinowich, "A method of phase shifting in the presence of vibration," in *ESO Proc.*, Garching, Germany, Vol. 30 (1988).
15. P. L. Wizinowich, "Phase shifting interferometry in the presence of vibration: A new algorithm and system," *Appl. Opt.* **29**, 3271–3279 (1990).
16. J. C. Wyant, "Phase shifting interferometry," http://www.optics.arizona.edu/jcwyant (1998).
17. D. Malacara, Ed., *Optical Shop Testing*, Wiley, New York (1992).
18. D. C. Ghiglia and M. D. Pritt, *Two-Dimensional Phase Unwrapping: Theory, Algorithms, and Software*, Wiley, New York (1998).
19. S. Zhang and P. S. Huang, "Novel method for structured light system calibration," *Opt. Eng.* **45**, 083601 (2006).

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