

# Gravitational wave memory in de Sitter spacetime

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## Abstract

We examine gravitational wave memory in the case where sources and detector are in an expanding cosmology. For simplicity, we treat the case where the cosmology is de Sitter spacetime, and discuss the possibility of generalizing our results to the case of a more realistic cosmology. We find results very similar to those of gravitational wave memory in an asymptotically flat spacetime, but with the magnitude of the effect multiplied by a redshift factor.

## INTRODUCTION

Gravitational wave memory, a permanent displacement of the gravitational wave detector after the wave has passed, has been known since the work of Zel'dovich and Polnarev [1], extended to the full nonlinear theory of general relativity by Christodoulou [2], and treated by several authors [3–15]. As is usual in the treatment of isolated systems, all these works consider asymptotically flat spacetimes. However, we do not live in an asymptotically flat spacetime, but rather in an expanding universe. For sources of gravitational waves whose distance from the detector is small compared to the Hubble radius, modeling the system as an asymptotically flat spacetime should be sufficient. However, some of the most powerful sources of gravitational waves (e.g. the collision of two supermassive black holes following the merger of their two host galaxies) are at cosmological distances where the asymptotically flat treatment is not sufficient.

In this paper we will treat gravitational wave memory in an expanding universe. To avoid the complications of the full nonlinear Einstein equations, our treatment will use perturbation theory. There is a well developed theory of cosmological perturbations (see e.g. the textbook treatment in [16]). However, this standard cosmological perturbation theory uses metric perturbations, and we have found [11] that the properties of gravitational memory are made more clear when using a manifestly gauge invariant perturbation theory based on the Weyl tensor. Cosmological perturbation theory using the Weyl tensor was developed by Hawking [17]. We will use a treatment similar to that of [17], but also, using the conformal flatness of Friedman-Lemaître-Robertson-Walker (FLRW) spacetimes, a treatment that draws heavily on the techniques used in [11].

Cosmological perturbations depend on the equation of state of the matter. The universe, both at the current time and at any previous times from which a realistic source of gravitational wave memory could come, is dominated by dust and a cosmological constant. For simplicity, in this treatment we will treat only the case of a cosmological constant, leaving the more general case of dust and a cosmological constant for subsequent work. Thus this work treats gravitational waves in an expanding de Sitter spacetime. The perturbation equations are developed in section II, the cosmological memory effect is calculated in section III, and the implications of the results are discussed in section IV.

## EQUATIONS OF MOTION

From the Bianchi identity  $\nabla_{[\epsilon} R_{\alpha\beta]\gamma\delta} = 0$  we have

$$g^{\epsilon\alpha}\nabla_{\epsilon}C_{\alpha\beta\gamma\delta} = \nabla_{[\gamma}S_{\delta]\beta} \quad (1)$$

where  $S_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{6}Rg_{\alpha\beta}$  and  $R = g^{\alpha\beta}R_{\alpha\beta}$ . Using the Einstein field equation with cosmological constant

$$R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta} + \Lambda g_{\alpha\beta} = 8\pi T_{\alpha\beta} \quad (2)$$

we find that eqn. (1) becomes

$$g^{\epsilon\alpha}\nabla_{\epsilon}C_{\alpha\beta\gamma\delta} = 8\pi\nabla_{[\gamma}X_{\delta]\beta} \quad (3)$$

Where  $X_{\alpha\beta} = T_{\alpha\beta} - \frac{1}{3}Tg_{\alpha\beta}$  and  $T = g^{\alpha\beta}T_{\alpha\beta}$ .

Both the Weyl tensor, and  $T_{\alpha\beta}$  vanish in de Sitter spacetime. It then follows that when we perturb eqn. (3) from a de Sitter background, the perturbed equation takes the same form with the Weyl tensor and stress-energy replaced by their (gauge invariant) perturbations and the metric and derivative operator replaced with their background values. We will rewrite this perturbed equation in a convenient form making use of the conformal flatness of de Sitter spacetime. Recall that the line element in a spatially flat Friedman-Lemaître-Robertson-Walker (FLRW) spacetime takes the form

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2) \quad . \quad (4)$$

Then introducing the usual conformal time  $\eta$  by  $\eta \equiv \int dt/a$ , we find that the line element takes the form

$$ds^2 = a^2 \left[ -d\eta^2 + dx^2 + dy^2 + dz^2 \right] \quad . \quad (5)$$

That is, the de Sitter metric takes the form  $g_{\alpha\beta} = a^2\eta_{\alpha\beta}$  where  $\eta_{\alpha\beta}$  is the Minkowski metric with Cartesian coordinates  $(\eta, x, y, z)$ . It then follows that the perturbed eqn. (3) takes the form

$$\partial^{\alpha} \left( a^{-1}C_{\alpha\beta\gamma\delta} \right) = 8\pi \left[ a\partial_{[\gamma}X_{\delta]\beta} + X_{\beta[\gamma}\partial_{\delta]}a + \eta_{\beta[\gamma}X_{\delta]\lambda}\partial^{\lambda}a \right] \quad . \quad (6)$$

Here  $\partial_{\alpha}$  is the coordinate derivative operator with respect to the Cartesian coordinates  $(\eta, x, y, z)$ . Also here and in what follows we use the convention that indicies are raised and lowered with the Minkowski metric  $\eta_{\alpha\beta}$ .

Following the method of [11] we now decompose all quantities in terms of spatial tensors as follows, using latin letters for spatial indicies:

$$E_{ab} \equiv a^{-1} C_{a\eta b\eta} \quad (7)$$

$$B_{ab} \equiv (a^{-1})^{\frac{1}{2}} \epsilon^{ef} {}_a C_{efb\eta} \quad (8)$$

$$\mu = T_{\eta\eta} \quad (9)$$

$$q_a = T_{\eta a} \quad (10)$$

$$U_{ab} = T_{ab} \quad (11)$$

Here  $\epsilon_{abc} = \epsilon_{\eta abc}$  where  $\epsilon_{\alpha\beta\gamma\delta}$  is the Minkowski spacetime volume element. Then eqn. (6) yields two constraint equations

$$\partial^b E_{ab} = 4\pi a \left( \frac{1}{3} \partial_a (2\mu + U^c{}_c) - \partial_\eta q_a \right) \quad (12)$$

$$\partial^b B_{ab} = 4\pi a \epsilon^{ef} {}_a \partial_e q_f \quad (13)$$

and two equations of motion

$$\begin{aligned} & \partial_\eta E_{ab} - \frac{1}{2} \epsilon_a{}^{cd} \partial_c B_{db} - \frac{1}{2} \epsilon_b{}^{cd} \partial_c B_{da} \\ &= 4\pi a \left[ \partial_{(a} q_{b)} - \frac{1}{3} \delta_{ab} \partial_c q^c - \partial_\eta (U_{ab} - \frac{1}{3} \delta_{ab} U^c{}_c) \right] + 4\pi a' (U_{ab} - \frac{1}{3} \delta_{ab} U^c{}_c) \end{aligned} \quad (14)$$

$$\partial_\eta B_{ab} + \frac{1}{2} \epsilon_a{}^{cd} \partial_c E_{db} + \frac{1}{2} \epsilon_b{}^{cd} \partial_c E_{da} = 2\pi a \left( \epsilon_a{}^{cd} \partial_c U_{db} + \epsilon_b{}^{cd} \partial_c U_{da} \right) \quad (15)$$

Here  $a' = da/d\eta$  and  $\delta_{ab}$ , the Kronecker delta, is the spatial metric of Minkowski spacetime.

We now want to decompose the spatial tensors into tensors on the two-sphere. We introduce the usual spherical polar coordinates  $(r, \theta, \phi)$  with the usual relation to the Cartesian coordinates  $(x, y, z)$ . We use capital latin letters to denote two-sphere components. From the electric part of the Weyl tensor  $E_{ab}$  we obtain a scalar  $E_{rr}$  as well as a vector and a symmetric, trace-free tensor given by

$$X_A = E_{Ar} \quad (16)$$

$$\tilde{E}_{AB} = E_{AB} - \frac{1}{2} H_{AB} E_C{}^C \quad (17)$$

Here  $H_{AB}$  is the metric on the unit two-sphere, and all two-sphere indicies are raised and lowered with this metric. Similarly, the decomposition of the magnetic part of the Weyl tensor yields  $B_{rr}$  and

$$Y_A = B_{Ar} \quad (18)$$

$$\tilde{B}_{AB} = B_{AB} - \frac{1}{2} H_{AB} B_C{}^C \quad (19)$$

The decomposition of the spatial vector  $q_a$  yields a two-sphere scalar  $q_r$  and vector  $q_A$ , while the decomposition of the spatial tensor  $U_{ab}$  yields two-sphere scalars  $U_{rr}$  and  $N \equiv U^c{}_c$ , vector  $V_A \equiv U_{Ar}$  and a symmetric trace-free tensor

$$W_{AB} = U_{AB} - \frac{1}{2}H_{AB}U^C{}_C \quad (20)$$

Then the constraint equations (eqns. (12) and (13)) become

$$\partial_r E_{rr} + 3r^{-1}E_{rr} + r^{-2}D^A X_A = 4\pi a \left( \frac{1}{3}\partial_r(2\mu + N) - \partial_\eta q_r \right) \quad (21)$$

$$\partial_r B_{rr} + 3r^{-1}B_{rr} + r^{-2}D^A Y_A = 4\pi a r^{-2} \epsilon^{AB} D_A q_B \quad (22)$$

$$\begin{aligned} \partial_r X_A + 2r^{-1}X_A - \frac{1}{2}D_A E_{rr} + r^{-2}D^B \tilde{E}_{AB} \\ = 4\pi a \left( \frac{1}{3}D_A(2\mu + N) - \partial_\eta q_A \right) \end{aligned} \quad (23)$$

$$\partial_r Y_A + 2r^{-1}Y_A - \frac{1}{2}D_A B_{rr} + r^{-2}D^B \tilde{B}_{AB} = 4\pi a \epsilon_A{}^B (D_B q_r - \partial_r q_B) \quad (24)$$

Here  $D_A$  is the derivative operator and  $\epsilon_{AB}$  is the volume element of the unit two-sphere.

The evolution equations (eqns. (14) and (15)) become

$$\partial_\eta B_{rr} + r^{-2} \epsilon^{AB} D_A X_B = 4\pi a r^{-2} \epsilon^{AB} D_A V_B \quad (25)$$

$$\begin{aligned} \partial_\eta E_{rr} - r^{-2} \epsilon^{AB} D_A Y_B \\ = 4\pi a \left( \partial_r q_r - \partial_\eta U_{rr} + \frac{1}{3}\partial_\eta(N - \mu) \right) + 4\pi a' \left( U_{rr} - \frac{1}{3}(2N + \mu) \right) \end{aligned} \quad (26)$$

$$\begin{aligned} \partial_\eta Y_A + \frac{1}{2}r^{-2} \epsilon^{CD} D_C \tilde{E}_{DA} + \frac{1}{4}\epsilon_A{}^C (3D_C E_{rr} - 2\partial_r X_C) \\ = 2\pi a \left( \epsilon_A{}^C \left( \frac{1}{2}D_C(3U_{rr} - N) - \partial_r V_C \right) + r^{-2} \epsilon^{BC} D_B W_{CA} \right) \end{aligned} \quad (27)$$

$$\begin{aligned} \partial_\eta X_A - \frac{1}{2}r^{-2} \epsilon^{CD} D_C \tilde{B}_{DA} - \frac{1}{4}\epsilon_A{}^C (3D_C B_{rr} - 2\partial_r Y_C) \\ = 2\pi a (D_A q_r + \partial_r q_A) - 4\pi a (r^{-1} q_A + \partial_\eta V_A) + 4\pi a' V_A \end{aligned} \quad (28)$$

$$\begin{aligned} \partial_\eta \tilde{B}_{AB} + \frac{1}{2}\epsilon_A{}^C (D_C X_B + r^{-1} \tilde{E}_{CB} - \partial_r \tilde{E}_{CB}) \\ + \frac{1}{2}\epsilon_B{}^C (D_C X_A + r^{-1} \tilde{E}_{CA} - \partial_r \tilde{E}_{CA}) + \frac{1}{2}H_{AB} \epsilon^{CD} D_C X_D \\ = 2\pi a \epsilon_A{}^C (D_C V_B + r^{-1} W_{CB} - \partial_r W_{CB}) \\ + 2\pi a \epsilon_B{}^C (D_C V_A + r^{-1} W_{CA} - \partial_r W_{CA}) + 2\pi a H_{AB} \epsilon^{CD} D_C V_D \end{aligned} \quad (29)$$

$$\begin{aligned} \partial_\eta \tilde{E}_{AB} - \frac{1}{2}\epsilon_A{}^C (D_C Y_B - \partial_r \tilde{B}_{CB} + r^{-1} \tilde{B}_{CB}) \\ - \frac{1}{2}\epsilon_B{}^C (D_C Y_A - \partial_r \tilde{B}_{CA} + r^{-1} \tilde{B}_{CA}) - \frac{1}{2}H_{AB} \epsilon^{CD} D_C Y_D \\ = 4\pi a \left( D_{(A} q_{B)} - \frac{1}{2}H_{AB} D_C q^C \right) - \partial_\eta W_{AB} + 4\pi a' W_{AB} \end{aligned} \quad (30)$$

## CALCULATION OF MEMORY

We now consider the behavior of the fields at large distances from the source. Unlike the asymptotically flat case, we cannot make use of the formal definition of null infinity: de Sitter conformal infinity is spacelike, and all gravitational radiation is negligible there. Instead we define the optical scalar  $u = \eta - r$  and consider the case of large  $r$  and moderate values of  $u$ . Note that in this case “large  $r$ ” means large compared to the wavelength of the gravitational waves emitted by the source, but *not* large compared to the Hubble length. That is, we treat the case where  $ra'$  is of order 1. In the case of Minkowski spacetime, it is shown in [11] that stress-energy gets to large  $r$  and moderate  $u$  by traveling in null directions: that is, the dominant component of the stress-energy takes the form

$$T_{\alpha\beta} = A\partial_\alpha u\partial_\beta u \quad (31)$$

We will assume that eqn. (31) also holds in our case. From conservation of stress-energy, it then follows that  $A$  takes the form  $A = La^{-2}r^{-2}$  where  $L$  is a function of  $u$  and the two-sphere coordinates. In physical terms, the quantity  $L$  is the power radiated per unit solid angle. That is we have

$$\mu = N = U_{rr} = -q_r = La^{-2}r^{-2} + \dots \quad (32)$$

with all other components of the stress-energy falling off more rapidly. Here  $\dots$  means “terms higher order in  $r^{-1}$ ” We assume that, as in the asymptotically flat case, the electric and magnetic parts of the Weyl tensor behave as follows:

$$\tilde{E}_{AB} = e_{AB}r + \dots \quad (33)$$

$$\tilde{B}_{AB} = b_{AB}r + \dots \quad (34)$$

$$X_A = x_A r^{-1} + \dots \quad (35)$$

$$Y_A = y_A r^{-1} + \dots \quad (36)$$

$$E_{rr} = Pr^{-3} + \dots \quad (37)$$

$$B_{rr} = Qr^{-3} + \dots \quad (38)$$

Here the coefficient tensor fields are functions of  $u$  and the two-sphere coordinates. Furthermore, we assume that, as in the asymptotically flat case, in the limit as  $|u| \rightarrow \infty$  the only one of these coefficient tensor fields that does not vanish is  $P$ . Note that because of

the relation between Cartesian and spherical coordinates  $\tilde{E}_{AB}$  behaving like  $r$  corresponds to Cartesian components of the electric part of the Weyl tensor behaving like  $r^{-1}$ .

Now keeping only the dominant terms in eqns. (21-24) and using the fact that  $ra'$  is of order unity, we obtain

$$-\dot{P} + D^A x_A = -8\pi L a^{-2}(a + ra') \quad (39)$$

$$-\dot{Q} + D^A y_A = 0 \quad (40)$$

$$-\dot{x}_A + D^B e_{AB} = 0 \quad (41)$$

$$-\dot{y}_A + D^B b_{AB} = 0 \quad (42)$$

Here an overdot means derivative with respect to  $u$ . Similarly, keeping only the dominant terms in eqns. (25-30) yields

$$\dot{Q} + \epsilon^{AB} D_A x_B = 0 \quad (43)$$

$$\dot{P} - \epsilon^{AB} D_A y_B = 8\pi L a^{-2}(a + ra') \quad (44)$$

$$\dot{y}_A + \frac{1}{2}\epsilon^{CD} D_C e_{DA} + \frac{1}{2}\epsilon_A^C \dot{x}_C = 0 \quad (45)$$

$$\dot{x}_A - \frac{1}{2}\epsilon^{CD} D_C b_{DA} - \frac{1}{2}\epsilon_A^C \dot{y}_C = 0 \quad (46)$$

$$\dot{b}_{AB} + \epsilon_A^C \dot{e}_{CB} = 0 \quad (47)$$

$$\dot{e}_{AB} - \epsilon_A^C \dot{b}_{CB} = 0 \quad (48)$$

From here, the analysis proceeds essentially as in [11]. By convention, the scale factor  $a$  is unity at the present time. Therefore at the position of the detector,  $\eta$  is the same as the usual time,  $E_{ab}$  (despite the factor of  $a^{-1}$  in eqn. (7)) is equal to the physical electric part of the Weyl tensor and thus is directly related to tidal force, and  $a'$  is equal to  $H_0$ , the Hubble constant. Note that eqn. (48) is redundant, since it is equivalent to eqn. (47). Since  $e_{AB}$  and  $b_{AB}$  vanish as  $u \rightarrow -\infty$ , it follows from eqn. (47) that  $b_{AB} = -\epsilon_A^C e_{CB}$ . This can be used to eliminate  $b_{AB}$  from eqns. (42) and (46) which then become

$$\dot{y}_A + \epsilon^{CD} D_C e_{DA} = 0 \quad (49)$$

$$\dot{x}_A - \frac{1}{2} D^C e_{CA} - \frac{1}{2} \epsilon_A^C \dot{y}_C = 0 \quad (50)$$

Combining eqn. (49) with eqn. (45) then yields

$$\dot{y}_A + \epsilon_A^B \dot{x}_B = 0 \quad (51)$$

However, since  $x_A$  and  $y_A$  vanish as  $u \rightarrow -\infty$ , it then follows from eqn. (51) that

$$y_A = -\epsilon_A^B x_B \quad (52)$$

Thus, we can eliminate  $y_A$  from eqns. (40) and (44) which then become

$$\dot{Q} + \epsilon^{AB} D_A x_B = 0 \quad (53)$$

$$\dot{P} - D^A x_A = 8\pi L(1 + rH_0) \quad (54)$$

But these equations are then redundant, since they are equivalent to eqns. (43) and (39) respectively. Thus the only independent quantities are  $e_{AB}$ ,  $x_A$ ,  $P$ ,  $Q$  and  $L$ . These quantities satisfy the following equations

$$D^B e_{AB} = \dot{x}_A \quad (55)$$

$$\epsilon^{BC} D_B e_{CA} = \epsilon_A^C \dot{x}_C \quad (56)$$

$$D_A x^A = \dot{P} - 8\pi L(1 + rH_0) \quad (57)$$

$$\epsilon^{AB} D_A x_B = -\dot{Q} \quad (58)$$

Now let's consider how to use eqns. (55-58) to find the memory. Recall that  $e_{AB}$  is (up to a factor involving the distance and the initial separation) the second time derivative of the separation of the masses. Thus we want to integrate  $e_{AB}$  twice with respect to  $u$ . Define the velocity tensor  $v_{AB}$ , memory tensor  $m_{AB}$  and a tensor  $z_A$  by

$$v_{AB} \equiv \int_{-\infty}^u e_{AB} du \quad (59)$$

$$m_{AB} \equiv \int_{-\infty}^{\infty} v_{AB} du \quad (60)$$

$$z_A \equiv \int_{-\infty}^{\infty} x_A du \quad (61)$$

Now consider two masses in free fall whose initial separation is  $d$  in the  $B$  direction. Then after the wave has passed they will have an additional separation. Call the component of that additional separation in the  $A$  direction  $\Delta d$ . Then it follows from the geodesic deviation equation that

$$\Delta d = -\frac{d}{r} m^A_B \quad (62)$$

To find  $m_{AB}$  we first integrate eqns. (55) and (56) to obtain

$$D^B v_{AB} = x_A \quad (63)$$

$$\epsilon^{BC} D_B v_{CA} = \epsilon_A^C x_C \quad (64)$$

Then integrating again from  $-\infty$  to  $\infty$  we obtain

$$D^B m_{AB} = z_A \tag{65}$$

$$\epsilon^{BC} D_B m_{CA} = \epsilon_A^C z_C \tag{66}$$

Now integrating eqns. (57) and (58) from  $-\infty$  to  $\infty$  yields

$$D_A z^A = \Delta P - 8\pi F(1 + rH_0) \tag{67}$$

$$\epsilon^{AB} D_A z_B = 0 \tag{68}$$

where the quantities  $\Delta P$  and  $F$  are defined by  $\Delta P = P(\infty) - P(-\infty)$  and  $F = \int_{-\infty}^{\infty} L du$ . In physical terms,  $F$  is the amount of energy radiated per unit solid angle. In deriving eqn. (68) we have used the fact that  $Q$  vanishes in the limit as  $|u| \rightarrow \infty$ . Since  $z_A$  is curl-free, there must be a scalar  $\Phi$  such that  $z_A = D_A \Phi$ . Then using eqns. (67) and (65) we find

$$D_A D^A \Phi = \Delta P - 8\pi F(1 + rH_0) \tag{69}$$

$$D^B m_{AB} = D_A \Phi \tag{70}$$

## DISCUSSION

We now consider the physical implications of these results, and in particular of eqns. (69-70). As in the asymptotically flat case, there are two kinds of gravitational wave memory: an ordinary memory due to sources that do not get out to infinity and a null memory due to sources that do get out to infinity. The ordinary memory is sourced by  $\Delta P$ , that is the change in the radial component of the electric part of the Weyl tensor. The null memory is sourced by  $F$ , the energy per unit solid angle radiated to infinity. However, in contrast to the asymptotically flat case, there is a factor of  $1 + rH_0$  multiplying  $F$ . Note that in cosmology, the wavelength of light from distant sources is redshifted by a factor of  $1 + z$  and that to first order in  $z$  we have  $1 + z = 1 + rH_0$ . Thus, expressed in terms of  $F$  and  $r$  it seems that the null memory is enhanced by a factor of  $1 + z$ . However,  $r$  is not a directly observed property of a distant object: instead we observe the luminosity of the object and infer a luminosity distance  $d_L$  related to  $r$  by  $d_L = r(1 + z)r$ . Since the observed memory is given by eqn. (62) which has a factor of  $r^{-1}$ , it follows that when expressed in terms of  $F$  and  $d_L$ , the null memory is enhanced by a factor of  $(1 + z)^2$ . However,  $F$  is the energy radiated per unit solid angle as measured by the observer, who is at cosmological distance

from the source. Instead, one might want to calculate the local  $F$  (which we will call  $F_{\text{loc}}$ ) as measured by an observer who is sufficiently far from the source to be in its wave zone but still at a distance small compared to the Hubble radius. That is,  $F_{\text{loc}}$  is the  $F$  that source would have in Minkowski spacetime. Because the energy of radiation is diminished by a factor of  $1/(1+z)$ , it follows that  $F = F_{\text{loc}}/(1+z)$ . Therefore, when expressed in terms of  $F_{\text{loc}}$  and the luminosity distance, the memory is enhanced by a factor of  $1+z$ .

Finally, we consider the possible generalization of our memory calculation to the case where the background cosmology contains dust in addition to the cosmological constant. What makes eqn. (3) so simple in a de Sitter background is that the terms on the right hand side involve derivatives of the stress-energy, and that these derivatives vanish in de Sitter spacetime. However, as shown in [17], for fluid matter the source terms in the equation of motion for the Weyl tensor involve the shear of the fluid four-velocity, and the equation for the Weyl tensor must be supplemented with equations for the shear. Thus one obtains a set of coupled equations involving the Weyl tensor and the shear. Nonetheless, it is argued by Thorne [18] that for gravitational waves from realistic sources propagating in a cosmological spacetime, one can apply the geometric optics approximation in which the wavelength of the gravitational waves is small compared to all other length scales in the problem. The implication of this argument for the calculation of gravitational wave memory in the cosmological setting is that one expects all the extra terms in the equations due to shear to be negligible. This question is currently under study.

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