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On the Fundamental Group of Manifolds of Non-positive Curvature

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Abstract. We prove theorems on the structure of the fundamental group of a compact riemannian manifold of non-positive curvature. In particular, a conjecture of J. Wolf [*J. Differential Geometry*, **2**, 421–446 (1968)] is proved.

Let M be a compact riemannian manifold of non-positive curvature and Π_1 (M) its fundamental group. We wish to announce the following theorems; proofs will be found in a thesis under preparation.

THEOREM 1. *Let G be a solvable subgroup of the fundamental group* $\Pi_1(M)$. *Then G is of finite index over an abelian subgroup.*

In particular, we give an affirmative answer to a problem raised by J. Wolf[1] in the following corollary.

COROLLARY 1. *If* $\Pi_1(M)$ *is solvable, then M is flat.*

COROLLARY 2. *Every solvable subgroup of* Π_1 *is finitely generated.*

THEOREM 2. *Let G be a subgroup of* $\Pi_1(M)$. *If G contains a subnormal maximal abelian subgroup A, then G is of finite index over A.*

COROLLARY 1. *Suppose M is of strictly negative curvature. If a subgroup G of* $\Pi_1(M)$ *is subnormal over an abelian subgroup, then G is cyclic. Hence, every solvable subgroup of* $\Pi_1(M)$ *is cyclic.*

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After this paper was written, Professor J. Wolf informed me that he and Professor D. Gromoll have also obtained a proof of his conjecture.

[1] Wolf, J. A., "Growth of finitely generated solvable groups and curvature of riemannian manifolds," *J. Differential Geometry*, **2**, 421–446 (1968).