

Compactification in first order gravity

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The Kaluza-Klein compactification process is applied in five dimensions to Chern Simons gravity, for the anti-de Sitter and Poincaré groups, using the first order formalism. In this context some solutions are found and analyzed. Also, the conserved charges associated to the solutions are computed.

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I. INTRODUCTION

The Kaluza-Klein construction [1, 2] showed that four dimensional interactions could be understood as manifestations of an underlying higher dimensional gravity. In five dimensions the gravitational theory is defined over a manifold \mathcal{M}_5 , with the topology of $\mathcal{M}_5 = \mathcal{M}_4 \times S^1$ where \mathcal{M}_4 is a four dimensional manifold. This construction actually corresponds to study the *gravity* of a fiber bundle [3] where S^1 and \mathcal{M}_4 are the fiber and the base space respectively.

The geometry of \mathcal{M}_5 motivates the introduction of a coordinate system (x^μ, φ) , with x^μ the coordinates on \mathcal{M}_4 and $\varphi \in [0, 2\pi[$ in order to describe the S^1 . In principle, one should introduce a Fourier expansion in φ for every field on \mathcal{M}_5 , however only the lowest order will be considered here. With these coordinates the line element at *lower order in the fifth dimension* reads

$$ds_5^2 = (g_{\mu\nu}(x) + \Phi(x)^2 A_\mu(x) A_\nu(x)) dx^\mu dx^\nu + 2\Phi(x)^2 A_\mu(x) dx^\mu d\varphi + \Phi(x)^2 d\varphi^2, \quad (1)$$

$A_\mu(x)$ is identified with an electromagnetic field potential and $\Phi(x)$ with a scalar field on \mathcal{M}_4 .

On the other hand, the introduction of fermions into gravity drags the need of extending the metric gravity making manifest the presence of a local Lorentz group in the tangent space [4]. To achieve this one needs to introduce an orthonormal basis \tilde{e}^A , usually called vielbein, and a connection for the local Lorentz group, $\tilde{\omega}^{AB}$, called spin connection. If the spin connection is considered an independent field then a reformulation of gravity, called first order gravity, arises. This formulation has proven to be worthy on its own, beyond the presence of fermions.

In this work some aspects of compactification of first order gravity are addressed. In particular some solutions are shown as well as their analysis. Compactification of a first order theory of gravity differs from metric formalism and allows to visualize some aspects which are usually ignored, for instance, the presence of torsion in

\mathcal{M}_4 . Given that Einstein theory has been well studied within Kaluza-Klein construction this work concentrate mostly on Chern Simons (CS) gravities.

II. GRAVITY AND COMPACTIFICATION

To begin with the discussion, the five dimensional case, shown in Eq.(1), will be reanalyzed in the context of first order gravity. To obtain the metric (1) one can choose general fünfbein \tilde{e}^A , with $A = 0 \dots 3, 5$,

$$\tilde{e}^a = e^a(x) \text{ and } \tilde{e}^5 = \Phi(x)(A(x) + e^5) \quad (2)$$

with latin index $a = 0 \dots 3$, $\Phi(x)$ is a scalar field, $A(x)$ is a 1-form on \mathcal{M}_4 and $e^5 = d\varphi$.

To introduce a connection compatible with the vielbein above one have to consider that $\xi = \partial_\varphi$ is a Killing vector for Eq.(1). A Killing vector generates a Lorentz transformation with parameters $\Delta^{AB} = I_\xi \tilde{\omega}^{AB} - \tilde{E}^{AM} \tilde{E}^{BN} (\nabla_M \xi_N)$, however in this case, see Eq.(2), $\Delta^{AB}(\xi) = 0$. Using this result, the most general connection compatible with Eq. (1) is given by

$$\tilde{\omega}^{ab} = \omega^{ab}(x) + \psi^{ab}(x)e^5 \text{ and } \omega^{a5} = v^a(x) + p^a(x)e^5, \quad (3)$$

where $\psi^{ab}(x)$ and $p^a(x)$ are scalars and $\omega^{ab}(x)$ and $v^a(x)$ are a one-form respectively on \mathcal{M}_4 .

The vielbein (2) and the connection (3) determine the curvature $\tilde{R}^{AB} = d\tilde{\omega}^{AB} + \tilde{\omega}^A_C \tilde{\omega}^{CB}$, obtaining

$$\begin{aligned} \tilde{R}^{ab} &= (R^{ab} - v^a v^b) + (D(\psi^{ab}) + p^a v^b - p^b v^a) e^5, \\ \tilde{R}^{a5} &= D(v^a) + (Dp^a - \psi^a_b e^b) e^5, \end{aligned} \quad (4)$$

and the torsion two-form $\tilde{T}^A = d\tilde{e}^A + \tilde{\omega}^A_B \tilde{e}^B$

$$\begin{aligned} \tilde{T}^a &= (T^a - \Phi v^a A) - (\psi^a_b e^b - A\Phi p^a + v^a \Phi) e^5, \\ \tilde{T}^5 &= (d(\Phi A) - v_c e^c) + (d\Phi + p_c e^c) e^5. \end{aligned} \quad (5)$$

III. DEFINITIONS OF CHARGES

Because the Kaluza-Klein construction is a very particular geometry, one can address part of the analysis of charges on a general ground without considering a particular theory of gravity.

The form of the Lagrangian,

$$L_5 = (L_4(x) + dB_3(x)) \wedge d\varphi \quad (6)$$

guaranties that the Noether charges obtained in five dimensions from L_5 are connected with the Noether charges in four dimensions from $L_4 + dB_3$ by

$$Q = \int_{\partial\Sigma_3 \times S^1} *J_5 \equiv 2\pi \int_{\partial\Sigma_3} *J_4, \quad (7)$$

where Σ_3 represents a family of space-like surfaces that foliates \mathcal{M}_4 . In this way the effective action in four dimensions contains all the physics of five dimensions. After this remark it becomes straight to obtain the mass or the angular momenta of any solution of this theory as the Noether charges associated with Killing vectors on \mathcal{M}_4 .

Recalling that by construction $\xi = \partial_\varphi$ is a Killing vector, the analysis above can be extended to obtain the electric charge. In any electromagnetic theory, the electric charge can be obtained as the Noether charge associated with the gauge transformations, $Q(\lambda)$, whose gauge parameters, say $\lambda(x)$, can be smeared out at infinity, in this case at $\partial\Sigma_3$. Thus the electric charge is given by

$$\tilde{q} = \left(\frac{1}{\lambda_0} Q(\lambda) \right)_{\partial\Sigma_3} \quad (8)$$

with $\lambda(x)|_{\partial\Sigma_3} = \lambda_0$. On the other hand, The Kaluza-Klein construction Eqs.(2,3) is invariant under the transformation,

$$\begin{aligned} \varphi &\rightarrow \varphi + \lambda(x) \\ A(x) &\rightarrow A(x) + d\lambda(x), \end{aligned} \quad (9)$$

where one recognizes a gauge transformation of A . By noticing that the subset of gauge transformations useful for Eq.(8) coincides with the transformation generated by $\xi = \partial_\varphi$, one finally obtains

$$q_4 = \int_{\partial\Sigma_3 \times S^1} *J_5(\xi). \quad (10)$$

IV. EINSTEIN GRAVITY

The five dimensional (first order) Einstein Hilbert (EH) action reads

$$I_{EH} = \kappa_G \int_{\mathcal{M}_5} \tilde{R}^{AB} \tilde{e}^C \tilde{e}^D \tilde{e}^F \varepsilon_{AB C D F}. \quad (11)$$

It yields the equations of motion

$$\begin{aligned} \mathcal{E}_F &= \tilde{R}^{AB} \tilde{e}^C \tilde{e}^D \tilde{e}^F \varepsilon_{AB C D F} = 0, \\ \tilde{T}^A &= 0 \end{aligned} \quad (12)$$

It must be stressed that in first order gravity the vanishing of torsion is a consequence of the equations of motion.

The vanishing of torsion, $\tilde{T}^A = 0$, by Eq. (5) determines that

$$v^a = -E^{a\mu} \left(\frac{1}{2} \Phi F_{\mu\nu} dx^\nu + \partial_\mu \Phi A \right), \quad (13)$$

$$p_a = -E_a{}^\mu \partial_\mu \Phi \text{ and } \psi_{ab} = -\frac{1}{2} E_a{}^\mu E_b{}^\nu F_{\mu\nu}, \quad (14)$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. This last equation identifies A with a field potential.

Remarkably the four dimensional torsion (see Eq.(5)) not only does not vanish, but it actually reads

$$T^a = \Phi A v^a \Leftrightarrow K^{ab} = \Phi F^{ab} A, \quad (15)$$

where K^{ab} is the contorsion one form.

In metric formalism one can skip the presence of torsion because it can be completely understood in terms of an electromagnetic field, see Eq.(15). This feature is not surprising, in fact it is well established that under some particular conditions, as those given here, a torsion tensor can be rewritten as an effective electric field, although this have been proven to be false in general [5].

Φ can not be constant

After the replacement of conditions (13) and (14) - obtained from $\tilde{T}^A = 0$ - into the action (11) it becomes

$$I_{EH} = \kappa_G \int_{\mathcal{M}_4 \times S^1} \Phi \left(R + \frac{1}{4} \Phi^2 F_{\mu\nu} F^{\mu\nu} \right) \sqrt{g} d\varphi d^4x, \quad (16)$$

where R is the standard four dimensional Ricci scalar. Since this five dimensional action (16) is independent of φ one can integrate it out obtaining the effective action

$$I_{eff} = 2\pi \kappa_G \int_{\mathcal{M}_4} \Phi \left(R + \frac{1}{4} \Phi^2 F_{\mu\nu} F^{\mu\nu} \right) \sqrt{g} d^4x. \quad (17)$$

It must be stressed that the equations of motion obtained from this action reproduce the Einstein equations (12) after using the ansatz (2) and (3).

Observing the final expression (17) one could consider to take Φ constant, and so to obtain the standard Einstein Maxwell theory. However this breaks the equivalence between five and four dimensions, since a constant Φ , through $G^\varphi_\varphi = 0$, implies $F_{\mu\nu} = 0$, yielding a trivial result.

V. BEYOND EINSTEIN

In higher dimensions the premise of second order equation of motion for the metric does not restrict the action to EH. One the simplest extension gives rise to Lovelock

gravities [6, 7, 8]. Furthermore in odd dimensions there are subfamilies of Lovelock gravities which coincide with CS gravities and so non vanishing torsion solutions [9] exists. Unfortunately only a few non vanishing torsion solutions are known [10, 11] at present time. It is worth to stress that CS gravities are genuine gauge theories for (A)dS and Poincaré groups respectively.

The Poincaré CS action in five dimensions [12] is the so called *Gauss Bonnet* term and reads

$$I_p = \kappa_G \int_{\mathcal{M}_5} \tilde{R}^{AB} \tilde{R}^{CD} \tilde{e}^F \varepsilon_{AB C D F} + dB_4, \quad (18)$$

where $B_4(x)$ is boundary term to be fixed later. Its equations of motion are $\tilde{R}^{AB} \tilde{R}^{CD} \varepsilon_{AB C D F} = 0$ and $\tilde{R}^{AB} \tilde{T}^C \varepsilon_{AB C D F} = 0$.

Effective theory

Using the Kaluza-Klein ansatz in the five dimensional action (18) together with the vanishing of torsion one can compute an effective four dimensional action starting from Eq.(18). The effective action reads

$$I_{eff} = 2\pi\kappa_G \int_{\mathcal{M}_4} \Phi (\Omega^{ab} \Omega^{cd} + 4\rho^a \tau^{bc} e^d + 4\sigma^a \Omega^{bc} e^d) \varepsilon_{abcd} \quad (19)$$

where

$$\begin{aligned} \Omega^{ab} &= \hat{R}^{ab} - \frac{1}{2}\Phi^2 F^{ab} F - \frac{1}{4}\Phi^2 F^a{}_c F^b{}_d e^c e^d \\ \tau^{ab} &= -\frac{1}{2}\hat{D}(\Phi^2 F^{ab}) + \frac{1}{2}d\Phi F^{ab} + \partial^{[a}\Phi F^{b]}{}_c e^c \\ \rho^a &= \frac{1}{2}\hat{D}(\Phi F^a{}_b) e^b + \partial^a \Phi F \\ \sigma^a &= \frac{\hat{D}(\partial^a \Phi)}{\Phi} + \frac{1}{4}\Phi^2 F^a{}_c F^c{}_d e^d \end{aligned} \quad (20)$$

The $\hat{\cdot}$ s on the derivative indicates that they are torsionless derivatives on four dimensions, *i.e.*, the contorsion K^{ab} has been explicitly separated in the equations above.

The equations of motion, written in terms of the effective fields displayed above, Eqs.(20), are cumbersome, thus we chose not to write them down. The action (19) reproduces the five dimensional CS equations of motion. It is straightforward to prove that a constant Φ , just as before, implies the vanishing of $F_{\mu\nu}$.

VI. SOLUTION

In this section some solutions of CS gravities in five dimensions will be discussed. A solution of the Poincaré CS gravity with spherical symmetry in four dimensions is given by

$$\begin{aligned} e^0 &= N(r)dt, & e^1 &= \frac{1}{g(r)}dr, \\ e^2 &= rd\theta, & e^3 &= r \sin(\theta)d\phi, \\ \Phi &= \Phi(r), & A &= a(r)dt, \end{aligned} \quad (21)$$

where

$$\begin{aligned} \Phi(r) &= c_1 r \pm \sqrt{c_1^2 r^2 + c_2 r + c_3 + \frac{c_2}{2c_1}}, \\ N(r) &= g(r) = \sqrt{1 - \frac{8q^2 c_1^3}{(c_2^2 - 4c_1^2 c_3)} \left(\frac{d\Phi}{dr}\right)^{-1}}, \\ a(r) &= \frac{q}{\Phi^2}. \end{aligned} \quad (22)$$

In this solution one can recognize four arbitrary integration constants, which occurs because CS gravity has non linear equations of motion.

A. Analysis

The analysis of the four dimensional metric is best carried out using the variable $R = r + \frac{c_2}{2c_1^2}$. So the metric is written

$$ds_4^2 = -N(R)^2 dt^2 + \frac{1}{N(R)^2} dR^2 + \left(R - \frac{c_2}{2c_1^2}\right)^2 d\Omega^2 \quad (23)$$

$$\Phi(R) = c_1 \left(R + \text{sgn}(c_1) \sqrt{R^2 + \kappa}\right),$$

$$N(R) = \sqrt{1 + \frac{2q^2}{c_1 \kappa} \left(\frac{d\Phi}{dR}\right)^{-1}},$$

$$a(R) = \frac{q}{\Phi(R)^2} \quad (24)$$

with

$$\kappa = \frac{4c_1^2 c_3 - c_2^2}{4c_1^4}$$

The values $\kappa < 0$ and $c_2 > 0$ lead to naked singularities or a metric with the wrong signature everywhere, therefore they are dismissed from the physical spectrum. The case with $c_1 < 0$, $\kappa < 0$ and $c_2 = 0$ deserves some attention and will be analyzed in a subsequent section.

The case $\kappa > 0$, $c_1 > 0$ and $c_2 = 0$ leads to a solution which is regular everywhere and is asymptotically flat, *i.e.*,

$$\lim_{r \rightarrow \infty} R_{\alpha\beta}^{\mu\nu} = 0, \quad (25)$$

in four and five dimensions. This solution may be regarded as a soliton. Because of its regularity this case will analyzed in detail.

B. The definitions of charges

After realizing that the action (18) is a CS action for the Poincaré group, one can skip the long process of re-obtaining the Noether charges. The Noether currents associated with the Killing vectors of a CS theory have

been discussed in Ref.[13]. In five dimensions it is given by

$$* \mathbf{J}_5(\eta) = 6 d \left(\int_0^1 dt \langle (A_1 - A_0) F_t I_\eta A_t \rangle \right), \quad (26)$$

where η is a Killing vector. $F_t = dA_t + A_t \wedge A_t$ with $A_t = tA_1 + (1-t)A_0$. $\langle \rangle$ is the trace in the group. Here A_0 and A_1 are connections in the same fiber having the generic form

$$A = \frac{1}{2} \tilde{\omega}^{AB} J_{AB} + \tilde{e}^A P_A, \quad (27)$$

P_A and J_{AB} are the generators of the Poincaré group. The charges are computed using the background A_0 . It is worth to stress that background independent methods exists to calculate Noether charges for the CS-AdS gravity [13, 14] but they could not be trivially adapted to the Poincaré case. The definition of A_0 as a flat connection, determines B_4 in Eq.(18). In this way the Noether charges are given by

$$Q(\eta) = \int_{\partial \Sigma \times S^1} * \mathbf{J}_5(\eta). \quad (28)$$

The charges of the above solution are associated with the Killing vectors $\zeta = \partial_t$ and ∂_φ respectively. On the other hand A_0 will be fixed as the flat connection obtained from the geometry,

$$ds_{bg}^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin(\theta)^2 d\phi^2) + d\varphi^2,$$

which is a five dimensional Minkowski space with one of its direction wrapped up.

C. Mass and electric charge

The mass can be obtained from the five dimensional Noether charge by Eq.(7) associated with the Killing vector ζ using Eq.(28). To compute the Noether charge of the five dimensional CS theory is formally simpler than the analysis in four dimensions where the effective theory, Eq. (19), is not purely gravitational but it contains matter. For the case $\kappa > 0$, $c_1 > 0$ and $c_2 = 0$ the mass is given by

$$M = Q(\zeta) = 8\pi^2 \kappa_G c_1 \sqrt{1 + \frac{q^2}{c_1^2 \kappa}} \left(\frac{q^2}{c_1^2 \kappa} + 4 \left(\sqrt{1 + \frac{q^2}{c_1^2 \kappa}} \left(1 + \frac{9q^2}{4c_1^2 \kappa} \right) - 1 \right) \right) \quad (29)$$

One can check that this mass is positive. Because the mass vanishes for $q = 0$ this solution can be cast as a pure electromagnetic solution, where the mass M represents the *mass* of the electromagnetic field. This conjecture

seems to be confirmed by the asymptotic behavior of this solution, where

$$N(r)^2 \approx 1 + \frac{Q^2}{r^2} + \dots$$

reproducing the case $m = 0$ and $Q \neq 0$ in the Reissner-Nordström solution. One may speculate that another solution, one that asymptotically behaves as RN solution with $m \neq 0$, should exist. Unfortunately the equations of motion obtained from the action (19) do not allow an obvious extension of the solution above (21) to confirm this conjecture.

In a similar way, the electric charge can be obtained using Eq.(10), where in this case the current to be integrated is given by Eq.(26). After a straightforward computation the electric charge is given by

$$\tilde{q} = Q(\xi) = -\frac{96\pi^2 \kappa_G q^3}{c_1 \kappa}. \quad (30)$$

VII. ADS IN FOUR

As previously noticed, the $c_1 < 0$ case requires a deeper analysis. In this case the function $f(R)^2$ diverges at $R \rightarrow \infty$ as

$$f(R)^2 \sim 1 + \frac{3q^2}{c_1^2 \kappa} + \frac{4q^2}{c_1^2 \kappa^2} R^2 \quad (31)$$

giving rise to an effective cosmological constant in four dimensions.

For $\kappa > 0$ the solution is regular everywhere and there is no horizon. The solution with $\kappa < 0$ has a singularity at $R = \sqrt{-\kappa}$ as well as an horizon at

$$r_+ = \frac{c_1^2 \kappa + 2q^2}{|q|} \sqrt{\frac{-\kappa}{c_1^2 \kappa + q^2}} \quad (32)$$

for the range $-\frac{q^2}{c_1^2} < \kappa < 0$.

For all values of κ this solution has the same electric charge of the $c_1 > 0$ case, see Eq.(30).

Unfortunately, the mass $Q(\zeta)$, diverges and furthermore it may not be possible to find out a background which can subtracts these divergences. Even though such a background may exists still this would be odd, since for a theory invariant under the Poincaré group only a flat space can represent a proper background. Maybe a background independent method to calculate the mass would give a finite one [15].

Given these considerations, the $c_1 < 0$ case must be excluded from the physical spectrum in the solution above.

VIII. CONES

Poincaré CS gravity restricted to a vanishing torsion solutions is known to have solutions with conical singularities. In a certain way this a generalization of what

is well known to happen in 2+1 dimensions where the existence of black holes is only possible with a negative cosmological constant. This feature is shown by the famous BTZ solution whose $\Lambda \rightarrow 0$ limit yields a cone. For that reason the analysis of conical solutions in this Poincaré CS-KK model can be of interest.

One of those solutions is given by the same ansatz (21) with $N(r) = \alpha$, $g(r) = \beta$, $a(r) = a_0$ and $\Phi(r)$ arbitrary. This solution represents a scalar field, Φ , defined over a manifold with a conical singularity.

The arbitrariness of a field is not new for the CS gravity, and although it may seem odd it is a natural consequence of a higher power differential operator. In fact one must note that α and β are also arbitrary.

The arbitrariness of the scalar field can be fixed by requiring that the solution has a physical meaning. To fulfill this requirement Φ must be smooth near $r = 0$ and the mass associated to this solution be a finite one.

As expected the electric charge vanishes in this case since $a = a_0$. The mass, $M = Q(\zeta)$, is given by

$$M = 4\pi^2 \kappa_G \frac{d\Phi(r)}{dr} \beta(\beta - 1) (5 + 7\alpha + 3\beta(1 + 3\alpha)), \quad (33)$$

which is finite provided

$$\lim_{r \rightarrow \infty} \Phi(r) \approx c_p r + c_q + O\left(\frac{1}{r}\right),$$

with c_p and c_q constants.

This result constraints the arbitrariness of Φ .

Comparing the above result Eq.(33) with the previous solution Eq.(22) one finds that $\beta = 1$ is equivalent to $q = 0$. That both solutions above share a sub-sector it probably indicates that there is a more general solution that includes both as particular cases.

IX. ADS IN FIVE

The introduction of a cosmological constant, $\Lambda \neq 0$, into a compactification procedure is not straightforward. First, one has to consider that if a natural ground state for $\Lambda = 0$ is a flat space with a non vanishing cosmological constant the ground state is expected to be a constant curvature manifold. An ansatz of the form $AdS_4 \times S^1$ don't fulfill this condition. Roughly speaking, this implies that the fifth dimension in Eq.(1) needs a *warp* factor, represented by a non constant Φ , even for the ground state.

The description of a fiber bundle by a constant curvature manifold can be difficult. One needs to isolate a cycle in \mathcal{M}_5 which can be identified with the fiber. Fortunately for the case of negative curvature manifolds there are a plethora of known spaces obtained as identifications of AdS which have a cycle by construction.

The extension of Einstein gravity with a negative cosmological constant is direct. For this reason only the AdS

CS gravity will be discussed. This gravity is given by

$$I_p = \kappa_G \int_{\mathcal{M}_5} \left(\tilde{R}^{AB} \tilde{R}^{CD} \tilde{e}^F + \frac{2}{3l^2} \tilde{R}^{AB} \tilde{e}^C \tilde{e}^D \tilde{e}^F + \frac{1}{5l^4} \tilde{e}^A \dots \tilde{e}^F \right) \varepsilon_{ABCD} + dB_4, \quad (34)$$

where B_4 is a boundary term to be defined later. The corresponding equations of motion are $\tilde{R}^{AB} \tilde{R}^{CD} \varepsilon_{ABCD} = 0$ and $\tilde{R}^{AB} \tilde{T}^C \varepsilon_{ABCD} = 0$, where

$$\tilde{\tilde{R}}^{AB} = \tilde{R}^{AB} + \frac{1}{l^2} \tilde{e}^A \tilde{e}^B.$$

The negative cosmological constant is given by $\Lambda = -6l^{-2}$. Black hole solutions for AdS CS theory can be found in [16].

A solution

For simplicity one can consider to turn off the electromagnetic field as a first approximation. In this case, to be consistent with the fiber bundle geometry, $\mathcal{M}_5 = \mathcal{M}_4 \times S^1$, the spherical transverse section in (21) must be replaced by a flat transverse section. After considering this simplification a solution is given by

$$\begin{aligned} \Phi(r) &= C_1 \left(\sqrt{3\gamma + 3\frac{r^2}{l^2}} - \frac{r}{l} \right), \\ g(r) &= \sqrt{\gamma + \frac{r^2}{l^2}} \\ N(r) &= C_1 \left(\sqrt{3\gamma + 3\frac{r^2}{l^2}} - \frac{r}{l} \right) \\ a(r) &= A_0 \\ \tilde{e}^2 &= rd\theta \\ \tilde{e}^3 &= rd\phi. \end{aligned} \quad (35)$$

This solution has an horizon at

$$r_+ = l \sqrt{-\frac{3}{2}\gamma}$$

provided that $\gamma < 0$. This solution has no meaning for $r < r_+$ ($N(r)$ becomes complex). The inner region ($r < r_+$) must be described by another chart. There is also a curvature singularity at $r = r_+$ but this is not a problem because this is a light-like surface thus there is no outgoing radiation. This solution, choosing $A_0 = 0$ and $\gamma = 0$, corresponds to the wormhole found in [18] with a Ricci flat base manifold.

The temperature of the four dimensional induced solution vanishes. Finally, as expected, this solution is asymptotically locally AdS, *i.e.*,

$$\lim_{r \rightarrow \infty} R_{\alpha\beta}^{\mu\nu} = -\frac{1}{l^2} \delta_{\alpha\beta}^{\mu\nu}. \quad (36)$$

The charges associated with this solution can be computed by using Eq.(26), in this case, for the AdS group. The background in this case corresponds to a locally AdS space. For simplicity it is considered the space described by the following metric

$$\begin{aligned} g(r) &= \frac{r}{l} & N(r) &= C_1 \left(\sqrt{3} - 1 \right) \frac{r}{l} \\ a(r) &= A_0, & \Phi(r) &= C_1 \left(\sqrt{3} - 1 \right) \frac{r}{l}. \end{aligned} \quad (37)$$

The presence of C_1 is only a matter of convention. It avoids dealing with the $(\sqrt{3} - 1)$ coefficient.

As expected the electric charge, $Q(\xi)$, vanishes in this case, since the field potential is constant. On the other hand the mass $Q(\zeta)$, is given by

$$M = -6\sqrt{3}\pi\kappa_G \frac{\gamma^2 C_1^2}{l^2} V_2, \quad (38)$$

where V_2 is the volume of the spatial transverse section described by (θ, ϕ) .

X. CONCLUSIONS AND PROSPECTS

In this work the procedure of compactification is reviewed within the context of first order gravity. The manifest presence of four dimensional torsion, disguised as an electric field, was well established by Einstein himself, but in first order gravity it becomes transparent.

Unfortunately CS gravities possess a complex phase space, therefore many interesting solutions that one could expect to exist are not obvious. In this article some indirect evidence of the existence of a solution with asymptotically Reissner-Nordström behavior have been found.

This is very promising since this probably indicates that CS gravity, a truly gauge theory, reproduces Einstein gravity at long distances in four dimensions.

The introduction of a negative cosmological constant was also analyzed. The results in this case are far more complex to analyze. For simplicity the solution is constructed as perturbation over a well known ground space [17], which induces a flat transverse section in four dimensions. The solution displayed is an extremal black hole whose mass can be negative for a certain range of the parameters. The existence of *negative mass* solutions is related with the presence of a negative cosmological constant, and can not be obviously ruled out. As for $\Lambda = 0$ its extension to the non extremal case is not obvious mainly because of the non linearity of the equations of motion.

Torsion introduces some new degrees of freedom which has been ignored in this work, however this is a very interesting direction to continue with this investigation. The research for non vanishing torsion solutions into CS gravities has proven to be a hard task. There is, however, direct evidence [11] that a non vanishing torsion could give room to reproduce standard solutions within CS gravity, in this case standard four dimensional solutions.

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