A Novel Factorial Adjustment Method for Testing Divisibility by 2 and 5

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Abstract

This note presents two original and creative divisibility tests for the integers 2 and 5. Both rules use the factorial of the last digit of a number, combined with a small correction when the factorial value would disrupt the required modular congruence. The resulting tests are mathematically equivalent to the classical last-digit rules but reach the same conclusion through an unconventional factorial-based approach. Each theorem includes a short proof sketch, an explanation of the mechanism, and four worked examples. The methods are valid precisely because 2 and 5 divide the base 10, allowing the last digit to fully determine the remainder modulo these divisors. The work is intended as an accessible piece of recreational number theory for educational purposes.

Keywords: factorial divisibility test, last-digit congruence, recreational number theory

1 Introduction

Standard divisibility rules for small primes and composites are usually based on simple digit patterns or weighted sums. In this short paper we introduce two novel tests that intentionally incorporate the factorial function $n!$ as the primary adjustment tool, with minimal overrides for the exceptional cases of $0!$ and $1!$. These rules were developed through exploratory modular arithmetic and refined with organizational assistance from Grok AI. The constructions are fully rigorous yet pedagogically engaging, and they highlight how a seemingly complicated operation can be adapted to reproduce classical results.

2 The Divisibility Rule for 5

Theorem 2.1 (Factorial Last-Digit Test for Divisibility by 5). *Let N be any non-negative integer and let $b = N \mod 10$ be its last digit. Define the adjustment function*

$$a(b) = \begin{cases} 0 & \text{if } b = 0, \\ b! & \text{otherwise.} \end{cases}$$

Form $S = N + a(b)$. Then

$$5 \mid N \iff 5 \mid S.$$

Proof sketch. Because $10 \equiv 0 \pmod 5$, it follows that $N \equiv b \pmod 5$. - If $b = 5$, then $5! = 120 \equiv 0 \pmod 5$, so $S \equiv b + 0 \equiv 0 \pmod 5$. - If $b = 0$, $0! = 1 \not\equiv 0 \pmod 5$; setting $a(0) = 0$ yields $S = N \equiv 0 \pmod 5$. - For $b \in \{1, 2, 3, 4, 6, 7, 8, 9\}$, exhaustive checking shows that $S \equiv 0 \pmod 5$ occurs precisely when $b \equiv 0 \pmod 5$ (i.e., only $b = 5$ in this range after correction). Hence the equivalence holds for all N.

Mechanism. The factorial $b!$ is a multiple of 5 exactly when $b \geq 5$. The only problematic case is $b = 0$ (where $0! = 1$), which is corrected by forcing the adjustment to zero. This restores $S \equiv N \pmod 5$ in all cases.

Examples:

1. $N = 35$, $b = 5$, $a = 120$, $S = 155$ (ends in 5) $\to$ divisible by 5 ($35 \div 5 = 7$).
2. $N = 12345$, $b = 5$, $a = 120$, $S = 12465$ (ends in 5) $\to$ divisible by 5 ($12345 \div 5 = 2469$).
3. $N = 67890$, $b = 0$, $a = 0$, $S = 67890$ (ends in 0) $\to$ divisible by 5 ($67890 \div 5 = 13578$).
4. $N = 111111$, $b = 1$, $a = 1$, $S = 111112$ (ends in 2) $\to$ not divisible by 5.

3 The Divisibility Rule for 2

Theorem 3.1 (Factorial Last-Digit Test for Divisibility by 2). *With the same notation, define*

$$a(b) = \begin{cases} 0 & \text{if } b = 0 \text{ or } b = 1, \\ b! & \text{otherwise.} \end{cases}$$

Form $S = N + a(b)$. Then

$$2 \mid N \iff 2 \mid S.$$

Proof sketch. Since $10 \equiv 0 \pmod 2$, we have $N \equiv b \pmod 2$. - For $b \geq 2$, $b!$ includes the factor 2 and is therefore even. - For $b = 0$ and $b = 1$, $0! = 1$ and $1! = 1$ are both odd; setting $a(b) = 0$ (even) avoids parity flip. - Thus $a(b)$ is always even, so $S \equiv N \pmod 2$ holds exactly.

Mechanism. Factorials of integers ≥ 2 are even by construction. The two exceptional cases ($0!$ and $1!$) are both odd and are corrected to zero to preserve parity equivalence.

Examples:

1. $N = 246$, $b = 6$, $a = 720$ (even), $S = 966$ (even) $\to$ divisible by 2 ($246 \div 2 = 123$).

2. $N = 1234567890$, $b = 0$, $a = 0$, $S = 1234567890$ (even) $\rightarrow$ divisible by 2.

3. $N = 3141592653$, $b = 3$, $a = 6$ (even), $S = \ldots 659$ (odd) $\rightarrow$ not divisible by 2.

4. $N = 1000000000000000002$, $b = 2$, $a = 2$, $S = \ldots 004$ (even) $\rightarrow$ divisible by 2.

4 Conclusion

The two rules illustrate that the factorial function—typically seen as combinatorial—can be cleverly adapted to divisibility testing with only minor case corrections. Both tests are fully equivalent to the well-known last-digit criteria for 2 and 5, and they are valid precisely because these divisors divide the base 10. The constructions offer an engaging classroom or self-study exploration of modular arithmetic and creative problem solving.

5 References

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