Yagah Prime Density Triangle: A Mathematical Framework Deriving Nuclear Magic Numbers and Nuclear Subshell Structure

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Abstract

We introduce the Yagah prime density triangle, a multiplicative array constructed by a modified rule of indices. Two display formats are given: a right-angle triangle with converted and unconverted entries, and an equilateral triangle obtained by mirroring the right-angle triangle about the central term x^n. We derive the row-sum generating function and provide worked examples using both direct computation and the generating function. Using a four-rule division scheme applied to the unconverted rows we obtain the sequence whose integer parts sum to the magic numbers for two spin orientations. Replacing the repeated end integers by 1,1 yields whose integer parts give the standard nuclear magic numbers $2, 8, 20, 28, 50, 82, 126, 184, \ldots$ Subshell filling up to x^9, deductions, and evidence from nuclear physics are included. A recurrence $M = m \pm [(n_1 n_2) + 2]$ is presented for generating further magic numbers. The sequences appear as OEIS A005897 and OEIS A018226.

1 Introduction

Triangular arrays such as Pascal's triangle, triangular numbers, and tetrahedral numbers encode combinatorial information through simple recurrences. Other known figurate numbers, including Pascal's triangle, triangular numbers, and tetrahedral numbers, are all derived from the Yagah prime density triangle. The full derivation and the broader framework are contained in a separate manuscript by the author.

In this paper we introduce the Yagah prime density triangle, a multiplicative array built by a modified rule of indices. Using a four-rule division scheme on the unconverted rows, we obtain two integer sequences.

The first sequence

$$0, 2, 6, 14, 28, 50, 82, 126, 184, \ldots$$

appears as OEIS A005897, known as the doubly even magic numbers.

The second sequence

$$2, 8, 20, 28, 50, 82, 126, 184, \ldots$$

appears as OEIS A018226, the standard nuclear magic numbers.

2 Yagah Prime Density Triangle

Let $x = 2$. The triangle is built by the modified rule: each front digit carries exponent 1, back numbers follow base-ten.

$$(1\times 2) \to 1^1\cdot 2^0 = 1 = x^0$$
$$(1\times 2)+(2\times 3) \to x^2+2$$
$$(1\times 2)+(2\times 3)+(3\times 4) \to x^3+6^2+3$$
$$(1\times 2)+(2\times 3)+(3\times 4)+(4\times 5) \to x^4+6^3+12^2+4$$
$$(1\times 2)+(2\times 3)+(3\times 4)+(4\times 5)+(5\times 6) \to x^5+6^4+12^3+20^2+5$$

Let $T(n,k)$ denote the k-th term in row n, $k=1,\dots,n+1$, $n\geq 0$.

2.1 Right-Angle Triangle Format

Unconverted figures:

x^0									
x^2	2								
x^3	6^2	3							
x^4	6^3	12^2	4						
x^5	6^4	12^3	20^2	5					
x^6	6^5	12^4	20^3	30^2	6				
x^7	6^6	12^5	20^4	30^3	42^2	7			
x^8	6^7	12^6	20^5	30^4	42^3	56^2	8		
x^9	6^8	12^7	20^6	30^5	42^4	56^3	72^2	9	

Converted figures:

1									
4	2								
8	36	3							
16	216	144	4						
32	1296	1728	400	5					
64	7776	20736	8000	900	6				
128	46656	248832	160000	27000	1764	7			
256	279936	2985984	3200000	810000	74088	3136	8		
512	1679616	35831808	64000000	24300000	3111696	175616	5184	9	

2.2 Equilateral Triangle Format (Pyramid)

		x^0						
		2	x^2	2				
		3	6^2	x^3	6^2	3		
	4	12^2	6^3	x^4	6^3	12^2	4	
5	20^2	12^3	6^4	x^5	6^4	12^3	20^2	5

Row n has the form:

$$n,\ [n(n+1)]^2,\ \dots,\ 6^3,\ 6^2,\ x^{(n+1)},\ 6^2,\ 6^3,\ \dots,\ [n(n+1)]^2,\ n.$$

3 Row Sum and Generating Function

The row sum is

$$S(r) = x^{(r+1)} + \sum_{j=2}^{r}[j(j+1)]^{(r-j+2)} + (r+1), \quad \text{with } x=2.$$

Generating function:

$$G(t) = \sum_{r\geq 0} S(r)t^r = \frac{1}{1-xt} + \sum_{j\geq 2}\frac{[j(j+1)]^2 t^{j-1}}{1-[j(j+1)]t} + \frac{t}{(1-t)^2}.$$

Worked examples:
A. Direct computation

- $r = 2$: $S(2) = x^3 + 6^2 + 3 = 8 + 36 + 3 = 47$.
- $r = 3$: $S(3) = x^4 + 6^3 + 12^2 + 4 = 16 + 216 + 144 + 4 = 380$.
- $r = 4$: $S(4) = x^5 + 6^4 + 12^3 + 20^2 + 5 = 32 + 1296 + 1728 + 400 + 5 = 3461$.

B. Using the generating function For $r = 2$, the coefficient of t^2 in $G(t)$ comes from:

1. $\frac{1}{1-2t}$ gives $2^2 = 4$ for the x^3 term.
2. $\frac{6^2 t}{1-6t}$ gives 36 for the 6^2 term.
3. $\frac{t}{(1-t)^2}$ gives 3 for the $+3$ term.

Summing: $4 + 36 + 3 = 47 = S(2)$.

For $r = 3$, the coefficient of t^3 is: $2^3 + 216 + 144 + 3 = 8 + 216 + 144 + 3 = 380 = S(3)$.

4 Derivation Through Yagah Row Division

Define four rules for division:

1. $\dfrac{x^{(k+1)}}{x^k} = x$.
2. $\dfrac{a_m}{a_{m-1}} = a$.
3. For the penultimate numerator with the last denominator, use b/c.
4. Leave the last numerator unchanged.

Applying these to unconverted rows gives sequence $\mathcal{D}$:

$$\begin{aligned}
\mathcal{D}_0 &= x^0 \\
\mathcal{D}_1 &= x^2 + 2 \\
\mathcal{D}_2 &= x + 3 + 3 \\
\mathcal{D}_3 &= x + 6 + 4 + 4 \\
\mathcal{D}_4 &= x + 6 + 12 + 5 + 5 \\
\mathcal{D}_5 &= x + 6 + 12 + 20 + 6 + 6 \\
\mathcal{D}_6 &= x + 6 + 12 + 20 + 30 + 7 + 7 \\
\mathcal{D}_7 &= x + 6 + 12 + 20 + 30 + 42 + 8 + 8 \\
\mathcal{D}_8 &= x + 6 + 12 + 20 + 30 + 42 + 56 + 9 + 9 \\
\mathcal{D}_9 &= x + 6 + 12 + 20 + 30 + 42 + 56 + 72 + 10 + 10
\end{aligned}$$

Worked examples of the division rule:

- For $\mathcal{D}_3$: Start with $x^4 + 6^3 + 12^2 + 4$. This gives $x + 6 + 4 + 4$.
- For $\mathcal{D}_4$: Start with $x^5 + 6^4 + 12^3 + 20^2 + 5$. This gives $x + 6 + 12 + 5 + 5$.

5 Magic Numbers for Two Spin Orientations

Adding the integer parts in $\mathcal{D}$ gives

$$0, 2, 6, 14, 28, 50, 82, 126, 184, 258, 350, 462, 596, 754, 938, 1150, \ldots$$

These are the magic numbers for two spin orientations.

Replace the repeated end integers by 1,1 to get $\mathcal{D}'$:

$$\begin{aligned} x^0 &\to x^0 \\ x^2+2 &\to x^2+1+1 \\ x+3+3 &\to x+1+1 \\ x+6+4+4 &\to x+6+1+1 \\ x+6+12+5+5 &\to x+6+12+1+1\ldots \end{aligned}$$

Summing gives $2, 8, 20, 28, 50, 82, 126, 184, \ldots$

Any magic number for the two-spin orientation can be obtained from

$$M = m \pm [(n_1 \cdot n_2) + 2],$$

where m is a known magic number and n_1, n_2 are the n-th positions.

Example: To get the 4th magic number after 14, take $m = 14$, $n_1 = 3$, $n_2 = 4$: $M = 14 + [(3 \times 4) + 2] = 28$.

The author predicts this fits the maize seed nucleon model, where nucleons pack in a tetrahedral geometry.

6 Subshell Filling up to x^9

Use notation $A(s/p)B$, where A is the number of nucleon groups, B nucleons per group, $A \times B$ total nucleons. $x^0 = 1$ is the fundamental nucleon state.

$\mathcal{D}_1$: $x^2 + 2 = 6$ $n(2) \to 1s^2 = 2$; $p(2) \to 2p^2 = 4$. Total 6.

$\mathcal{D}_2$: $x + 3 + 3 = 8$ $n(2) \to 1s^2 = 2$; $p(3,3) \to 2p^3 = 6$. Total 8.

$\mathcal{D}_3$: $x + 6 + 4 + 4 = 16$ $x = n(2) \to 1s^2 = 2$; $6 = 2p(3) \to 2p^3 = 6$; $4 + 4 = 2n(4) \to 3s^4 = 8$. Total 16.

$\mathcal{D}_4$: $x + 6 + 12 + 5 + 5 = 30$ $x = n(2) \to 1s^2 = 2$; $6 = 2p(3) \to 2p^3 = 6$; $12 = 3n(4) \to 3s^4 = 12$; $5 + 5 = 2p(5) \to 2p^5 = 10$. Total 30.

$\mathcal{D}_5$: $x + 6 + 12 + 20 + 6 + 6 = 52$ $x = n(2) \to 1s^2 = 2$; $6 = 2p(3) \to 2p^3 = 6$; $12 = 3n(4) \to 3s^4 = 12$; $20 = 4p(5) \to 4p^5 = 20$; $6 + 6 = 3n(6) \to 4s^6 = 12$. Total 52.

$\mathcal{D}_6$: $x + 6 + 12 + 20 + 30 + 7 + 7 = 84$ Fills to $4p^6 = 12$, $5s^6 = 12$, $3n(7) \to 5p^7 = 14$. Total 84.

$\mathcal{D}_7$: $x + 6 + 12 + 20 + 30 + 42 + 8 + 8 = 128$ Fills to $5p^6 = 12$, $6s^6 = 12$, $4n(8) \to 6p^8 = 16$. Total 128.

$\mathcal{D}_8$: $x + 6 + 12 + 20 + 30 + 42 + 56 + 9 + 9 = 186$ Fills to $6p^8 = 16$, $7s^6 = 12$, $5n(9) \to 7p^9 = 18$. Total 186.

$\mathcal{D}_9$: $x + 6 + 12 + 20 + 30 + 42 + 56 + 72 + 10 + 10 = 260$ Fills to $7p^8 = 16$, $8s^6 = 12$, $6n(10) \to 8p^{10} = 20$. Total 260.

Neutrons have odd base and even exponent; protons have the opposite. This mirrors isospin symmetry.

7 Deductions

1. The "s" shell corresponds to standing waves without spin; the "p" shell corresponds to standing waves with three orientations x, y, z. Completed shells have total angular momentum $J = 0$.

2. At the magic numbers the nucleus is in the island of stability.

3. In the island of stability the nucleons are deformed, with $2s, 2p$ adding shells while keeping nucleon numbers intact.

4. Neutrons have odd base and even exponent; protons have the opposite.

5. The author predicts this scenario best fits the maize seed nucleon model.

8 Evidence Supporting Magic Numbers in Neutron Shells

1. San Jose State University applet-magic.com, Thayer Watkins. Incremental binding energy shows shell closures at $2, 8, 20, 28, 50, 82, 126$. For two spin orientations: $2, 6, 14, 28, 50, 82, 126, 184$.
2. Xavier Borges, "Magic numbers derived from a variable phase nuclear model," *The General Science Journal*, 2005. Predicts magic number 184, which appears in the Yagah sequence.
3. Wikipedia, "Magic number (physics)". Double magic nuclei include ^{4}He, ^{16}O, ^{48}Ca, ^{56}Ni, ^{132}Sn, ^{208}Pb. Island of stability predicted around $Z = 114$–126, $N = 184$.
4. O. Haxel, J. H. D. Jensen, and H. E. Suess, "On the 'Magic Numbers' in Nuclear Structure," *Phys. Rev.* **75**, 1766 (1949).
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9 Conclusion

The Yagah prime density triangle provides a combinatorial construction whose row division yields the magic numbers for two spin orientations, and after replacing repeated end integers, the standard nuclear magic numbers $2, 8, 20, 28, 50, 82, 126, 184, \ldots$ The generating function for row sums and the recurrence $M = m \pm [(n_1 n_2) + 2]$ give explicit computational tools. The subshell filling and deductions align with a tetrahedral packing model for nucleons.

References

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