Prime Density Triangle: A Mathematical Framework Deriving Nuclear Magic Numbers, Shell Gaps and subshell structures

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Abstract

We introduce the prime density triangle, a multiplicative array constructed by a modified rule of indices. Two display formats are given: a right-angle triangle with converted and unconverted entries, and an equilateral triangle obtained by mirroring the right-angle triangle about the central term x^n. We derive the row-sum generating function and provide worked examples using both direct computation and the generating function. Using a four-rule division scheme applied to the unconverted rows we obtain the sequence whose integer parts sum to the magic numbers for two spin orientations. Replacing the repeated end integers by 1,1 yields integer parts that give the standard harmonic oscillator (HO) magic numbers and, in conjunction with the two-spin-orientation nucleon magic numbers, gives precisely the standard nuclear magic numbers $2, 8, 20, 28, 50, 82, 126, 184$. Furthermore, digital-root reduction of the converted triangle reveals a distinct "4,9,9" pattern whose diagonals concatenate into large primes, establishing a direct arithmetic link between prime distribution and the symmetry conditions of nuclear shell closures. Subshell filling up to x^9, deductions, and evidence from nuclear physics are included. A recurrence $M = m \pm [(n_1 n_2) + 2]$ is presented for generating further magic numbers. The sequences appear as OEIS A005897, OEIS A018226 and OEIS A007290.

1 Introduction

Triangular arrays such as Pascal's triangle, triangular numbers, and tetrahedral numbers encode combinatorial information through simple recurrences. Other known figurate numbers, including Pascal's triangle, triangular numbers (directly from D9/2), OEIS A000217 and tetrahedral numbers (Tn+D/2) OEIS A000292 are all derived from the prime density triangle.

In this paper we introduce the prime density triangle, a multiplicative array built by a modified rule of indices. Using a four-rule division scheme on the unconverted rows, we obtain two integer sequences.

The first sequence

$$0, 2, 6, 14, 28, 50, 82, 126, 184, \ldots$$

appears as OEIS A005897, known as the doubly even magic numbers.

The second sequence

$$2, 8, 20, 28, 50, 82, 126, 184, \ldots$$

appears as OEIS A018226, the standard nuclear magic numbers. A digital-root analysis of the converted entries further uncovers a structured prime-generating pattern, bridging number-theoretic prime distribution with the symmetry conditions required for nuclear stability.

2 Prime Density Triangle

The triangle is built by the following rule: each front digit carries exponent 1, back numbers follow base-ten, then exponents are added after multiplication. We set $X = 2$.

The construction for the first few rows (right-angle format) is:

$$(1 \times 2) \rightarrow 1^1 \times 2^0 = 1 = 2^0 = x^0,$$
$$(1 \times 2) + (2 \times 3) \rightarrow (1^1 \times 2^1) + (2^1 \times 3^0) = 2^2 + 2^1 = x^2 + 2,$$
$$(1 \times 2) + (2 \times 3) + (3 \times 4) \rightarrow (1^1 \times 2^2) + (2^1 \times 3^1) + (3^1 \times 4^0) = 2^3 + 6^2 + 3^1 = x^3 + 6^2 + 3,$$
$$(1 \times 2) + (2 \times 3) + (3 \times 4) + (4 \times 5) \rightarrow x^4 + 6^3 + 12^2 + 4,$$
$$(1 \times 2) + (2 \times 3) + (3 \times 4) + (4 \times 5) + (5 \times 6) \rightarrow x^5 + 6^4 + 12^3 + 20^2 + 5,$$

and so on. Let $T(n, k)$ denote the k-th term in row n with $k = 1, \ldots, n + 1$, $n \geq 0$.

2.1 Right-Angle Triangle Format

Unconverted figures:

$$\begin{aligned}
D_0 &= x^0 \\
D_1 &= x^2 + 2 \\
D_2 &= x + 3 + 3 \\
D_3 &= x + 6 + 4 + 4 \\
D_4 &= x + 6 + 12 + 5 + 5 \\
D_5 &= x + 6 + 12 + 20 + 6 + 6 \\
D_6 &= x + 6 + 12 + 20 + 30 + 7 + 7 \\
D_7 &= x + 6 + 12 + 20 + 30 + 42 + 8 + 8 \\
D_8 &= x + 6 + 12 + 20 + 30 + 42 + 56 + 9 + 9 \\
D_9 &= x + 6 + 12 + 20 + 30 + 42 + 56 + 72 + 10 + 10
\end{aligned}$$

Figure 1: Unconverted right-angle triangle format.

Converted figures (row sum, with $x = 2$):

$$D_0 = 1,\ D_1 = 6,\ D_2 = 8,\ D_3 = 16,\ D_4 = 30,\ D_5 = 52,\ D_6 = 84,\ D_7 = 128,\ D_8 = 186,\ D_9 = 260.$$

Figure 2: Converted figures (row sums with $x = 2$).

$*1$ **Primes from the rows.** Define two formulas that generate primes with high probability (with $x = 2$):

(1) For an integer prime or odd exponent p_1, $x^{p_1} \pm p_1$ yields prime numbers. Examples: $x^3 \pm 3 = 2^3 \pm 3 = 8 \pm 3 = 5$ and 11; $x^9 \pm 9 = 2^9 \pm 9 = 512 \pm 9 = 503$ and 521 (both primes).

(2) For other exponents p_o (non-prime integers, including even numbers), $x^{p_o} \pm (p_o - 1)$ yields primes. Examples: $x^4 \pm (4-1) = 2^4 \pm 3 = 16 \pm 3 = 13$ and 19; $x^2 \pm (2-1) = 2^2 \pm 1 = 4 \pm 1 = 3$ and 5. (For $p_o = 2$ this reproduces the first formula with $p_1 = 2$.)

$*2$ **Triangular and Tetrahedral numbers from the row divisions.** For Triangular numbers; take row 9, D9/2. Thus, $x + 6 + 12 + 20 + 30 + 42 + 56 + 72 - 2$ where $x = 2$. We get $1, 3, 6, 10, 15, 21, 28, 36$, which are exactly the triangular numbers T_n. For Tetrahedral numbers; dividing the 2 spin orientation magic numbers by 2 in terms then adding in terms the triangular numbers. For the first $2/2 = 1$, we maintain it. $M = 2, 6, 14, 28, 50, 82$, dividing by 2, it implies $2 \div 2 = 1$ (maintain), $6 \div 2 + 1(T1) = 4$, $14 \div 2 + 3(T2) = 10$, $28 \div 2 + 6(T3) = 20$, $50 \div 2 + 10(T4) = 35$. So $1, 4, 10, 20, 35, 56, 84, \ldots$ corresponds exactly to the Tetrahedral numbers (T_n).

2.2 Digital Root Reduction and the 4,9,9 Pattern

It is the **converted figures** from Section 2.1 to which we apply digital-root reduction. Taking each converted entry and persistently summing its digits until a single digit remains yields the digital-root triangle shown in Figure 4.

1
4 2
8 9 3
7 9 9 4
5 9 9 4 5
1 9 9 8 9 6
2 9 9 7 9 9 7

Figure 3: Digital-root triangle derived from the converted figures.

Reading the triples diagonally from this digital-root triangle gives the sequence shown in Table 1.

Table 1: Diagonal triples and their concatenated values.

Diagonal starting at	Triple	Concatenated
Row 1, Col 1	$4, 9, 9$	499
Row 2, Col 1	$8, 9, 9$	899
Row 3, Col 1	$7, 9, 9$	799
Row 4, Col 1	$1, 9, 9$	199
Row 5, Col 1	$5, 9, 9$	599
Row 6, Col 1	$2, 9, 9$	299

The sequence loops back to $4, 9, 9$. Arranging the concatenated triples in the order above yields:

$$499899799199599299.$$

Grouping these digits into larger constructs and testing for primality gives:

$$1998997995994992994 9,$$

$$199899799599499299799,$$

$$199899799599499299199,$$

all of which are prime numbers(Which We termed as the researcher's primes)

Interpretation and Link to Nuclear Stability: In digital-root arithmetic, the digit 9 represents completeness, symmetry, and a state of zero net angular momentum—precisely the conditions expected for a closed nuclear shell with total angular momentum $J = 0$. The overwhelming prevalence of 9 in the interior of the reduced triangle mirrors the stability of doubly magic nuclei. Meanwhile, the specific non-9 digits that survive on the perimeter arrange themselves into diagonals that concatenate into primes. This demonstrates that the same multiplicative array which generates the magic numbers also encodes a structured prime distribution at the digital-root level, providing an explicit arithmetic bridge between number theory and nuclear shell structure.

2.3 Equilateral Triangle Format (Pyramid)

Row n has the form:

$$n,\ [n(n+1)]^2, \ldots, 6^3, 6^2, x^{(n+1)}, 6^2, 6^3, \ldots, [n(n+1)]^2, n.$$

$$
\begin{array}{l}
0 \\
2 + X^2 + 2 \\
3 + 6^2 + X^3 + 6^2 + 3 \\
4 + 12^2 + 6^3 + X^4 + 6^3 + 12^2 + 4 \\
5 + 20^2 + 12^3 + 6^4 + X^5 + 6^4 + 12^3 + 20^2 + 5 \\
6 + 30^2 + 20^3 + 12^4 + 6^5 + X^6 + 6^5 + 12^4 + 20^3 + 30^2 + 6 \\
7 + 42^2 + 30^3 + 20^4 + 12^5 + 6^6 + X^7 + 6^6 + 12^5 + 20^4 + 30^3 + 42^2 + 7 \\
8 + 56^2 + 42^3 + 30^4 + 20^5 + 12^6 + 6^7 + X^8 + 6^7 + 12^6 + 20^5 + 30^4 + 42^3 + 56^2 + 8 \\
9 + 72^2 + 56^3 + 42^4 + 30^5 + 20^6 + 12^7 + 6^8 + X^9 + 6^8 + 12^7 + 20^6 + 30^5 + 42^4 + 56^3 + 72^2 + 9
\end{array}
$$

Figure 4: Equilateral triangle format (pyramid).

3 Row Sum and Generating Function

The row sum for the right angled triangle format is

$$S(r) = x^{(r+1)} + \sum_{j=2}^{r} [j(j+1)]^{(r-j+2)} + (r+1), \quad \text{with } x = 2.$$

Generating function:

$$G(t) = \sum_{r \geq 0} S(r)t^r = \frac{1}{1 - xt} + \sum_{j \geq 2} \frac{[j(j+1)]^2 t^{j-1}}{1 - [j(j+1)]t} + \frac{t}{(1-t)^2}.$$

Worked examples:
A. Direct computation

$$r = 2 : S(2) = x^3 + 6^2 + 3 = 8 + 36 + 3 = 47,$$
$$r = 3 : S(3) = x^4 + 6^3 + 12^2 + 4 = 16 + 216 + 144 + 4 = 380,$$
$$r = 4 : S(4) = x^5 + 6^4 + 12^3 + 20^2 + 5 = 32 + 1296 + 1728 + 400 + 5 = 3461.$$

B. Using the generating function For $r = 2$, the coefficient of t^2 in $G(t)$ comes from:

1. $\frac{1}{1-2t}$ gives $2^2 = 4$ for the x^3 term.

2. $\frac{6^2 t}{1-6t}$ gives 36 for the 6^2 term.

3. $\frac{t}{(1-t)^2}$ gives 3 for the $+3$ term.

Summing: $4 + 36 + 3 = 47 = S(2)$.
For $r = 3$, the coefficient of t^3 is: $2^3 + 216 + 144 + 3 = 8 + 216 + 144 + 3 = 380 = S(3)$.

4 Derivation Through Row Division

Define four rules for division:

1. $\frac{x^{(k+1)}}{x^k} = x$

2. $\frac{a_m}{a_{m-1}} = a$

3. For the penultimate numerator with the last denominator, use b/c

4. Leave the last numerator unchanged.

Applying these to unconverted rows gives sequence $\mathcal{D}$ (already shown above). Worked examples of the division rule:
For $\mathcal{D}_3$: Start with $\mathcal{D}_4/\mathcal{D}_3 = (x^4 + 6^3 + 12^2 + 4) \div (x^3 + 6^2 + 3)$. This gives $x + 6 + 4 + 4$.
For $\mathcal{D}_4$: Start with $\mathcal{D}_5/\mathcal{D}_4 = (x^5 + 6^4 + 12^3 + 20^2 + 5) \div (x^4 + 6^3 + 12^2 + 4)$. This gives $x + 6 + 12 + 5 + 5$.

5 Magic Numbers for Two Spin Orientations, Harmonic Oscillator, and Nucleon Magic Numbers

Adding the integer parts in $\mathcal{D}$ gives

$$0, 2, 6, 14, 28, 50, 82, 126, 184, 258, 350, 462, 596, 754, 938, 1150, \ldots$$

These are the magic numbers for two spin orientations. For example: (1) $x^0 + 0 = 0$, (2) $x^2 + 2 = 2$, (3) $x + 6 + 3 = 8$, (4) $x + 6 + 4 + 4 = 14$, (5) $x + 6 + 12 + 5 + 5 = 28$, (6) $x + 6 + 12 + 20 + 6 + 6 = 50$, etc.
Replacing the repeated end integers by 1, 1 gives the Harmonic oscillator (HO) magic numbers:

$$x^0 \to x^0, \quad x^2{+}2 \to x^2{+}1, \quad x{+}3{+}3 \to x{+}1{+}1, \quad x{+}6{+}4{+}4 \to x{+}6{+}1{+}1, \quad x{+}6{+}12{+}5{+}5 \to x{+}6{+}12$$

Adding the integer parts gives the HO magic numbers in the 3-dimensional isotropic oscillator model for nuclei (cumulative filled shells):

$$2, 8, 20, 40, 70, 112, 168, 240, \ldots$$

In conjunction with the two-spin-orientation magic numbers, replace 6, 14 in the latter set with 8, 20 to obtain the standard nucleon magic numbers:

$$2, 8, 20, 28, 50, 82, 126, 184, 258, 350, 462, 596, \ldots$$

Any magic number for the two-spin orientation can be obtained from

$$M = m \pm [(n_1 n_2) + 2],$$

where m is a known magic number and n_1, n_2 are the n-th positions.

Example: To get the 4th magic number after 14, take $m = 14$, $n_1 = 3$, $n_2 = 4$. Then $M = 14 + [(3 \times 4) + 2] = 28$. And with $n_1 = 2$, $n_2 = 3$, $M = 14 - [(2 \times 3) + 2] = 14 - 8 = 6$ (the magic number before 14 for the two-spin orientation). The author predicts this fits the maize seed nucleon model, where nucleons pack in a tetrahedral geometry.

6 Energy Calibration and Shell-Gap Predictions from Unconverted Row Sums

6.1 Row Sums and Magic Numbers

Table 2: Row sums $S(r)$ evaluated with $x = 2$, and the two-spin-orientation magic numbers $M(r)$.

Row r	Unconverted row (evaluated)	$S(r)$	$M(r)$
1	$2^2 + 6$	2	2
2	$2^3 + 6^2 + 3$	47	6
3	$2^4 + 6^3 + 12^2 + 4$	380	14
4	$2^5 + 6^4 + 12^3 + 20^2 + 5$	3461	28
5	$2^6 + 6^5 + 12^4 + 20^3 + 30^2 + 6$	37482	50
6	$2^7 + 6^6 + 12^5 + 20^4 + 30^3 + 42^2 + 7$	484387	82
7	$2^8 + 6^7 + 12^6 + 20^5 + 30^4 + 42^3 + 56^2 + 8$	7353408	126
8	$2^9 + 6^8 + 12^7 + 20^6 + 30^5 + 42^4 + 56^3 + 72^2 + 9$	129104441	184
9	$2^{10} + 6^9 + 12^8 + 20^7 + 30^6 + 42^5 + 56^4 + 72^3 + 90^2 + 10$	2589967502	258
10	$2^{11} + 6^{10} + 12^9 + 20^8 + 30^7 + 42^6 + 56^5 + 72^4 + 90^3 + 110^2 + 11$	58757627063	350

6.2 Energy Ordering Within a Row

In the unconverted row, energy lessens from left to right, directly tied to the decrease in exponents. For the row $2^5 + 6^4 + 12^3 + 20^2 + 5$, the highest energy is carried by 2^5 (exponent 5), followed by 6^4 (exponent 4), and so on, down to the constant term 5 (exponent 1). Energy decreases horizontally across the row. This energy ordering explains why deformation can occur without net energy loss: after row division the row ends with a repeated pair (e.g. $5 + 5$ or, after suitable replacement, $1 + 1$), keeping the total energy intact.

6.3 Calibration of the Shell-Gap Formula

The difference between successive row sums is

$$\Delta S(r) = S(r) - S(r-1) \quad (r \geq 2).$$

The experimental single-particle shell gaps at the major magic numbers that coincide with $M(r)$ are

$$M = 28: \ 6.0 \text{ MeV}, \qquad M = 50: \ 5.0 \text{ MeV}, \qquad M = 126: \ 4.0 \text{ MeV}.$$

A power-law relation

$$\Delta E(r) = C[\Delta S(r)]^{\alpha}[M(r)]^{\beta}$$

is fitted to the three calibration points

$$(M = 28, \Delta S = 3081), \quad (M = 50, \Delta S = 34021), \quad (M = 126, \Delta S = 6869021).$$

Solving the resulting system of equations gives

$$C = 22.3 \text{ MeV}, \qquad \alpha = 0.0456, \qquad \beta = -0.503.$$

Thus the shell-gap energy determined solely from the row-sum difference and the magic number is

$$\boxed{\Delta E(r) = 22.3[\Delta S(r)]^{0.0456}[M(r)]^{-0.503} \text{ MeV}}.$$

The formula reproduces the three calibration gaps exactly.

6.4 Predicted Shell Gaps

Table 3: Predicted shell-gap energies from the difference of consecutive row sums and the magic number.

$M(r)$	$\Delta S(r)$	ΔE_{model} (MeV)	Remarks
6	4	11.0	subshell
14	333	7.7	subshell
28	3081	6.0	doubly magic ^{48}Ca region
50	34021	5.0	doubly magic ^{100}Sn region
82	446905	4.41	$1h_{11/2}$ gap (exp. 4.5–5.0 MeV)
126	6869021	4.0	doubly magic ^{208}Pb
184	121751033	3.8	predicted island of stability
258	2460863061	3.7	superheavy island region
350	56167659561	3.62	predicted

6.5 Worked Examples

Example 1: $M = 28$ (row 4)

$$\Delta S(4) = 3461 - 380 = 3081, \qquad M(4) = 28.$$

$$[\Delta S]^{0.0456} = \exp(0.0456 \times \ln 3081) = \exp(0.0456 \times 8.0326) = \exp(0.3663) = 1.442,$$
$$[M]^{-0.503} = \exp(-0.503 \times \ln 28) = \exp(-0.503 \times 3.3322) = \exp(-1.676) = 0.1870,$$
$$\Delta E = 22.3 \times 1.442 \times 0.1870 = 22.3 \times 0.2697 = 6.02 \approx 6.0 \text{ MeV}.$$

Example 2: $M = 126$ (row 7)

$$\Delta S(7) = 7353408 - 484387 = 6869021, \qquad M(7) = 126.$$
$$[\Delta S]^{0.0456} = \exp(0.0456 \times \ln 6869021) = \exp(0.0456 \times 15.742) = \exp(0.7178) = 2.050,$$
$$[M]^{-0.503} = \exp(-0.503 \times \ln 126) = \exp(-0.503 \times 4.8363) = \exp(-2.433) = 0.0878,$$
$$\Delta E = 22.3 \times 2.050 \times 0.0878 = 22.3 \times 0.1800 = 4.01 \approx 4.0 \text{ MeV}.$$

Example 3: $M = 350$ (row 10)

$$\Delta S(10) = 58757627063 - 2589967502 = 56167659561, \qquad M(10) = 350.$$
$$[\Delta S]^{0.0456} = \exp(0.0456 \times \ln 56167659561) = \exp(0.0456 \times 24.7516) = \exp(1.1287) = 3.091,$$
$$[M]^{-0.503} = \exp(-0.503 \times \ln 350) = \exp(-0.503 \times 5.8579) = \exp(-2.9465) = 0.05255,$$
$$\Delta E = 22.3 \times 3.091 \times 0.05255 = 22.3 \times 0.1624 = 3.62 \text{ MeV}.$$

7 Subshell Filling up to x^9

7.1 Important Note on the Proposed Nucleon Subshell Filling

This refers exclusively to nuclear shells governed by the strong nuclear force, distinct from atomic electron shells. The "s" and "p" labels follow a classification derived from the Prime Density Triangle statistics.

7.2 Island of Stability and Deformation via Repeated End Terms (n, n)

A central feature of the model is the interpretation of the repeated end integers $(n + n)$ that appear at the close of each D row. These paired identical terms indicate paired nucleon groups that maintain structural stability. In the island of stability, nucleons do not lose energy when filling these levels. Instead, they become deformed while preserving the overall nucleon count. This deformation allows nucleons to be added to the next subshell without disrupting the closed-shell stability.

7.3 Explanation of Group Interpretation for Repeated End Pairs

For the central triangular terms the interpretation follows a straightforward multiplier × base pattern. However, for the repeated end pairs (n, n), the grouping is more nuanced and does not mean simple multiplication.

Example (D5: $6 + 6$): This is interpreted as $3n(6) \to 4s^6 = 12$. It is not 3×6 or 4×6 in the simple sense. Instead, it represents multiple groupings of the 4s orbital (here three groups), each associated with the repeated value 6, yielding the total of 12. The repeated ends reflect the paired/deformed nature in the island of stability.

Demonstration A: $x + 3 + 3$ (= 8 nucleons): Neutron: $n(2) \to 1s^2 = 2$, proton $p(3,3) \to 2p^3 = 6$. Total: 8.

Demonstration B: $x+6+12+5+5$ (= 30 nucleons): $x = n(2) \to 1s^2 = 2$; $6 = 2p(3) \to 2p^3 = 6$; $12 = 3n(4) \to 3s^4 = 12$; $5+5 = 2p(5) \to 2p^5 = 10$. Total 30. Use notation $A(s/p)B$, where A is the number of nucleon groups, B nucleons per group, $A \times B$ total nucleons. $x^0 = 1$ is the fundamental nucleon state.

D_1: $x^2 + 2 = 6$: $n(2) \to 1s^2 = 2$, $p(2) \to 2p^2 = 4$. Total 6. D_2: $x + 3 + 3 = 8$: $n(2) \to 1s^2 = 2$, $p(3,3) \to 2p^3 = 6$. Total 8. D_3: $x + 6 + 4 + 4 = 16$: $n(2) \to 1s^2 = 2$, $6 = 2p(3) \to 2p^3 = 6$, $4 + 4 = 2n(4) \to 3s^4 = 8$. Total 16. D_4: $x + 6 + 12 + 5 + 5 = 30$: $n(2) \to 1s^2 = 2$, $6 = 2p(3) \to 2p^3 = 6$, $12 = 3n(4) \to 3s^4 = 12$, $5+5 = 2p(5) \to 2p^5 = 10$. Total 30. D_5: $x + 6 + 12 + 20 + 6 + 6 = 52$: $n(2) \to 1s^2 = 2$, $6 = 2p(3) \to 2p^3 = 6$, $12 = 3n(4) \to 3s^4 = 12$, $20 = 4p(5) \to 4p^5 = 20$, $6 + 6 = 3n(6) \to 4s^6 = 12$. Total 52. D_6: $x + 6 + 12 + 20 + 30 + 7 + 7 = 84$: Fills to $4p^6 = 12$, $5s^6 = 12$, $3n(7) \to 5p^7 = 14$. Total 84. D_7: $x + 6 + 12 + 20 + 30 + 42 + 8 + 8 = 128$: Fills to $5p^6 = 12$, $6s^6 = 12$, $4n(8) \to 6p^8 = 16$. Total 128.

7.4 Complete Subshell Filling (Spin-Aware Version with p-Orbital Spins x, y, z)

From D2 onward, the spin (p orbital with x, y, z) version uses multiplication throughout from the beginning to the end.

1. $\mathbf{x^2 + 2}$: First shell takes 6 nuclides. $n(2,2) + p(2) \to 2s^2, 2p^1$. Neutron: $2s^2 = 4$, Proton: $2p^1 = 2$. Total: 6.
2. $\mathbf{x + 3 + 3}$: $n(2) + p(3,3) : 1s^2, 2p^3$.
3. $\mathbf{x + 6 + 4 + 4}$: $n(2) + 2p(3) + 2n(4) : 1s^2, 2s^4, 2p^3$.
4. $\mathbf{x + 6 + 12 + 5 + 5}$: $n2 + 3n(4) + 2p(3) + 2p(5) : 1s^2, 3s^4, 2p^2, 2p^5$.
5. $\mathbf{x + 6 + 12 + 20 + 6 + 6}$: $n2 + 3n(4) + 2n(6) + 2p(3) + 4p(5) : 1s^2, 3s^4, 2s^6, 2p^3, 4p^5_x$.
6. $\mathbf{x + 6 + 12 + 20 + 30 + 7 + 7}$: $n2+3n(4)+5n(6)+2p(3)+4p(5)+2p(7) : 1s^2, 3s^4, 5s^6, 2p^3, 4p^5_x, 2p^7_y$.
7. $\mathbf{x + 6 + 12 + 20 + 30 + 42 + 8 + 8}$: $n2 + 3n(4) + 5n(6) + 2n(8) + 2p(3) + 4p(5) + 6p(7) : 1s^2, 3s^4, 5s^6, 2s^8, 2p^3, 4p^5_x, 6p^7_y$.
8. $\mathbf{x + 6 + 12 + 20 + 30 + 42 + 56 + 9 + 9}$: $n2 + 3n(4) + 5n(6) + 7n(8) + 2p(3) + 4p(5) + 6p(7) + 2p(9) : 1s^2, 3s^4, 5s^6, 7s^8, 2p^3, 4p^5_x, 6p^7_y, 2p^9_x$.
9. $\mathbf{x + 6 + 12 + 20 + 30 + 42 + 56 + 72 + 10 + 10}$: continues the pattern.

Neutrons have odd base and even exponent; protons have the opposite. This mirrors isospin symmetry.

8 Deductions

1. The "s" shell corresponds to standing waves without spin; the "p" shell corresponds to standing waves with three orientations x, y, z. Completed shells have total angular momentum $J = 0$.

2. At the magic numbers the nucleus is in the island of stability.

3. In the island of stability the nucleons are deformed, with $2s, 2p$ adding shells while keeping nucleon numbers intact.

4. Neutrons have odd base and even exponent; protons have the opposite.

5. The author predicts this scenario best fits the maize seed nucleon model (or packing model as they stacked together).

9 Evidence Supporting Magic Numbers in Neutron Shells

1. San Jose State University applet-magic.com, Thayer Watkins. Incremental binding energy shows shell closures at $2, 8, 20, 28, 50, 82, 126$. For two spin orientations: $2, 6, 14, 28, 50, 82, 126, 184$.

2. Xavier Borges, "Magic numbers derived from a variable phase nuclear model," The General Science Journal, 2005. Predicts magic number 184, which appears in the sequence.

3. Wikipedia, "Magic number (physics)". Double magic nuclei include ^{4}He, ^{16}O, ^{48}Ca, ^{56}Ni, ^{132}Sn, ^{208}Pb. Island of stability predicted around $Z = 114 - 126$, $N = 184$.

4. O. Haxel, J. H. D. Jensen, and H. E. Suess, "On the 'Magic Numbers' in Nuclear Structure," Phys. Rev. 75, 1766 (1949).

5. M. G. Mayer, "On Closed Shells in Nuclei. II," Phys. Rev. 75, 1969-1970 (1949).

10 Conclusion

The prime density triangle provides a combinatorial construction whose row division yields the magic numbers for two spin orientations, and after replacing repeated end integers, the standard nuclear magic numbers $2, 8, 20, 28, 50, 82, 126, 184, \ldots$. The generating function for row sums and the recurrence $M = m \pm [(n_1 n_2) + 2]$ give explicit computational tools. The subshell filling and deductions align with a tetrahedral packing model for nucleons.

Data Availability Statement

The data and code that support the findings of this study are openly available in Zenodo at `https://doi.org/10.5281/zenodo.20233053` and in Mathscidoc at mathscidoc:2605.13001.

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