

Prime Density Triangle: A Mathematical Framework Deriving Nuclear Magic Numbers and Nuclear Subshell Structure

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Abstract

We introduce the a prime density triangle, a multiplicative array constructed by a modified rule of indices. Two display formats are given: a right-angle triangle with converted and unconverted entries, and an equilateral triangle obtained by mirroring the right-angle triangle about the central term x^n . We derive the row-sum generating function and provide worked examples using both direct computation and the generating function. Using a four-rule division scheme applied to the unconverted rows we obtain the sequence whose integer parts sum to the magic numbers for two spin orientations. Replacing the repeated end integers by 1,1 yields whose integer parts give the standard harmonic oscillator,HO and in conjunction with the 2 spin orientation nucleon magic numbers gives precisely the standard nuclear magic numbers 2, 8, 20, 28, 50, 82, 126, 184, ... Subshell filling up to x^9 , deductions, and evidence from nuclear physics are included. A recurrence $M = m \pm [(n_1 n_2) + 2]$ is presented for generating further magic numbers. The sequences appear as OEIS A005897 and OEIS A018226 and OEIS A007290

1 Introduction

Triangular arrays such as Pascal's triangle, triangular numbers, and tetrahedral numbers encode combinatorial information through simple recurrences. Other known figurate numbers, including Pascal's triangle, triangular numbers(directly from $D9/2$) and tetrahedral numbers($Tn+D/2$) are all derived from the Yagah prime fdensity triangle. The full derivation and the broader framework are contained in a separate manuscript by the author.

In this paper we introduce the Yagah prime density triangle, a multiplicative array built by a modified rule of indices. Using a four-rule division scheme on the unconverted rows, we obtain two integer sequences.

The first sequence

$$0, 2, 6, 14, 28, 50, 82, 126, 184, \dots$$

appears as OEIS A005897, known as the doubly even magic numbers.

The second sequence

$$2, 8, 20, 28, 50, 82, 126, 184, \dots$$

appears as OEIS A018226, the standard nuclear magic numbers.

2 Prime Density Triangle

The triangle is built by the modified rule: each front digit carries exponent 1, back numbers follow base-ten, then exponents added after multiplication. We set $X=2$

$$\begin{aligned}
(1 \times 2) &\rightarrow 1^1 \cdot 2^0 = 1 = x^0 \\
(1 \times 2) + (2 \times 3) &\rightarrow (1^1 X 2^1) + (2^1 X 3^0) = 2^2 + 4 = x^2 + 2 \\
(1 \times 2) + (2 \times 3) + (3 \times 4) &\rightarrow (1^1 X 2^2) + (2^1 X 3^1) + (3^1 X 4^0) = 2^3 + 6^2 + 3 = x^3 + 6^2 + 3 \\
(1 \times 2) + (2 \times 3) + (3 \times 4) + (4 \times 5) &\rightarrow x^4 + 6^3 + 12^2 + 4 \\
(1 \times 2) + (2 \times 3) + (3 \times 4) + (4 \times 5) + (5 \times 6) &\rightarrow x^5 + 6^4 + 12^3 + 20^2 + 5
\end{aligned}$$

Let $T(n, k)$ denote the k -th term in row n , $k = 1, \dots, n + 1$, $n \geq 0$.

2.1 Right-Angle Triangle Format

Unconverted figures:

$$\begin{array}{cccccccc}
x^0 & & & & & & & & \\
x^2 & 2 & & & & & & & \\
x^3 & 6^2 & 3 & & & & & & \\
x^4 & 6^3 & 12^2 & 4 & & & & & \\
x^5 & 6^4 & 12^3 & 20^2 & 5 & & & & \\
x^6 & 6^5 & 12^4 & 20^3 & 30^2 & 6 & & & \\
x^7 & 6^6 & 12^5 & 20^4 & 30^3 & 42^2 & 7 & & \\
x^8 & 6^7 & 12^6 & 20^5 & 30^4 & 42^3 & 56^2 & 8 & \\
x^9 & 6^8 & 12^7 & 20^6 & 30^5 & 42^4 & 56^3 & 72^2 & 9
\end{array}$$

*1 Primes from the rows. Let $p^1 = \text{integer odd/prime and } p^0 = \text{noninteger prime/odd}$

(1) Odd/prime exponents of x or its last term (which is the same as the exponent) generate primes of high probability, $x^{p^1} + \text{or} - p^1 = p^1$ Examples : $x^3 + \text{or} - 3 = 2^3 + \text{or} - = 8 + 3 = 11$ and $8 - 3 = 5(7, 5)$ both primes. Also $x^9 + \text{or} - 9 = 2^9 + 9 = 521$ and $2^9 - 9 = 503(521, 503)$ both primes. Other integer parts within the row also follow $\text{or} - p^0 + \text{or} - 1 = p^1$. Example : $x^2 + 2 = (2^2 + 2) = 6 + 1 = 7$, and $6 - 1 = 5$. Also, $(x^4 + 4) - 1 = 19$, $(x^4 - 4) = 12 + 1 = 13$, $12 - 1 = 11$

*2 Triangular and Tetrahedral numbers from the row divisions.

For Triangular numbers; taken the row 9, $D9/2$. Thus, $x+6+12+20+30+42+56+72 \div 2$ where $x=2$. We get 1,3,6,10,15,21,28,36,..., which are exactly the triangular numbers T_n . For Tetrahedral numbers; dividing the 2 spin orientation magic numbers by 2 in terms then adding in terms the triangular numbers. For the first $2/2=1$, we maintain it. $M=2,6,14, 28, 50, 82,..$ dividing by 2, it implies $2 \div 2=1$ (maintain), $6 \div 2+1(T1)=4$, $14 \div 2+3(T2)=10$, $28 \div 2+6(T3)=20$, $50 \div 2+10(T4)=35,..$ So 1,4,10,20,35,56,84,.. corresponds exactly the Tetrahedral numbers (Tr).

Converted figures:

$$\begin{array}{cccccccc}
1 & & & & & & & & \\
4 & 2 & & & & & & & \\
8 & 36 & 3 & & & & & & \\
16 & 216 & 144 & 4 & & & & & \\
32 & 1296 & 1728 & 400 & 5 & & & & \\
64 & 7776 & 20736 & 8000 & 900 & 6 & & & \\
128 & 46656 & 248832 & 160000 & 27000 & 1764 & 7 & & \\
256 & 279936 & 2985984 & 3200000 & 810000 & 74088 & 3136 & 8 & \\
512 & 1679616 & 35831808 & 64000000 & 24300000 & 3111696 & 175616 & 5184 & 9
\end{array}$$

2.2 Equilateral Triangle Format (Pyramid)

Row n has the form:

$$n, [n(n+1)]^2, \dots, 6^3, 6^2, x^{(n+1)}, 6^2, 6^3, \dots, [n(n+1)]^2, n.$$

$$\begin{array}{c}
0 \\
2+X^2+2 \\
3+6^2+X^3+6^2+3 \\
4+12^2+6^3+X^4+6^3+12^2+4 \\
5+20^2+12^3+6^4+X^5+6^4+12^3+20^2+5 \\
6+30^2+20^3+12^4+6^5+X^6+6^5+12^4+20^3+30^2+6 \\
7+42^2+30^3+20^4+12^5+6^6+X^7+6^6+12^5+20^4+30^3+42^2+7 \\
8+56^2+42^3+30^4+20^5+12^6+6^7+X^8+6^7+12^6+20^5+30^4+42^3+56^2+8 \\
9+72^2+56^3+42^4+30^5+20^6+12^7+6^8+X^9+6^8+12^7+20^6+30^5+42^4+56^3+72^2+9
\end{array}$$

Figure 1: Superimposing Yagah prime density triangle side by side

3 Row Sum and Generating Function

The row sum for the right angled triangle format is

$$S(r) = x^{(r+1)} + \sum_{j=2}^r [j(j+1)]^{(r-j+2)} + (r+1), \quad \text{with } x = 2.$$

Generating function:

$$G(t) = \sum_{r \geq 0} S(r)t^r = \frac{1}{1-xt} + \sum_{j \geq 2} \frac{[j(j+1)]^2 t^{j-1}}{1-[j(j+1)]t} + \frac{t}{(1-t)^2}.$$

Worked examples:

A. Direct computation

- $r = 2$: $S(2) = x^3 + 6^2 + 3 = 8 + 36 + 3 = 47$.
- $r = 3$: $S(3) = x^4 + 6^3 + 12^2 + 4 = 16 + 216 + 144 + 4 = 380$.
- $r = 4$: $S(4) = x^5 + 6^4 + 12^3 + 20^2 + 5 = 32 + 1296 + 1728 + 400 + 5 = 3461$.

B. Using the generating function For $r = 2$, the coefficient of t^2 in $G(t)$ comes from:

1. $\frac{1}{1-2t}$ gives $2^2 = 4$ for the x^3 term.
2. $\frac{6^2 t}{1-6t}$ gives 36 for the 6^2 term.
3. $\frac{t}{(1-t)^2}$ gives 3 for the $+3$ term.

Summing: $4 + 36 + 3 = 47 = S(2)$.

For $r = 3$, the coefficient of t^3 is: $2^3 + 216 + 144 + 3 = 8 + 216 + 144 + 3 = 380 = S(3)$.

4 Derivation Through Yagah Row Division

Define four rules for division:

1. $\frac{x^{(k+1)}}{x^k} = x$.
2. $\frac{a_m}{a_{m-1}} = a$.
3. For the penultimate numerator with the last denominator, use b/c .
4. Leave the last numerator unchanged.

Applying these to unconverted rows gives sequence \mathcal{D} :

$$\begin{aligned}\mathcal{D}_0 &= x^0 \\ \mathcal{D}_1 &= x^2 + 2 \\ \mathcal{D}_2 &= x + 3 + 3 \\ \mathcal{D}_3 &= x + 6 + 4 + 4 \\ \mathcal{D}_4 &= x + 6 + 12 + 5 + 5 \\ \mathcal{D}_5 &= x + 6 + 12 + 20 + 6 + 6 \\ \mathcal{D}_6 &= x + 6 + 12 + 20 + 30 + 7 + 7 \\ \mathcal{D}_7 &= x + 6 + 12 + 20 + 30 + 42 + 8 + 8 \\ \mathcal{D}_8 &= x + 6 + 12 + 20 + 30 + 42 + 56 + 9 + 9 \\ \mathcal{D}_9 &= x + 6 + 12 + 20 + 30 + 42 + 56 + 72 + 10 + 10\end{aligned}$$

Worked examples of the division rule:

- For \mathcal{D}_3 : Start with $D_4/D_3 = x^4 + 6^3 + 12^2 + 4 \div x^3 + 6^2 + 3$. This gives $x + 6 + 4 + 4$.
- For \mathcal{D}_4 : Start with $D_5/D_4 = x^5 + 6^4 + 12^3 + 20^2 + 5 \div x^4 + 6^3 + 12^2 + 4$. This gives $x + 6 + 12 + 5 + 5$.

5 Magic Numbers for Two Spin Orientations, Harmonic oscillator HO and Nucleon magic numbers

Adding the integer parts in \mathcal{D} gives

$$0, 2, 6, 14, 28, 50, 82, 126, 184, 258, 350, 462, 596, 754, 938, 1150, \dots$$

These are the magic numbers for two spin orientations. For example: (1) $x^0 + 0 = 0$. (2) $x^2 + 2 = 2$. (3) $x + 3 + 3 = 6$. (4) $x + 6 + 4 + 4 = 14$. (5) $x + 6 + 12 + 5 + 5 = 28$. (6) $x + 6 + 12 + 20 + 6 + 6 = 50$. Replacing the repeated end integers by 1, 1 to get \mathcal{D}' :

$$\begin{aligned}x^0 &\rightarrow x^0 \\ x^2 + 2 &\rightarrow x^2 + 1 \\ x + 3 + 3 &\rightarrow x + 1 + 1 \\ x + 6 + 4 + 4 &\rightarrow x + 6 + 1 + 1 \\ x + 6 + 12 + 5 + 5 &\rightarrow x + 6 + 12 + 1 + 1 \dots\end{aligned}$$

Adding the integer parts by summing gives the Harmonic oscillator magic numbers in the 3-dimensional isotropic oscillator model for nuclei, i.e., the cumulative filled shells. The HO magic numbers are:

$$2, 8, 20, 40, 70, 112, 168, 240, \dots$$

In conjunction of the HO magic numbers with the 2 spin orientation magic numbers, replace 6, 14 in the 2 spin orientation set with 8, 20. The new set for the 2 spin orientations becomes the standard nucleon magic numbers:

$$2, 8, 20, 28, 50, 82, 126, 184, (258, 350, 462, 596, \dots \text{the predicted next ones})$$

Any magic number for the two-spin orientation can be obtained from

$$M = m \pm [(n_1 n_2) + 2],$$

where m is a known magic number and n_1, n_2 are the n -th positions.

Example: To get the 4th magic number after 14, take $m = 14$, $n_1 = 3$, $n_2 = 4$: $M = 14 + [(3 \times 4) + 2] = 28$. And $n_1=2, n_2=3, M=14$, thus, $14 - [(2 \times 3) + 2] = 14 - 8 = 6$ (that's the magic number before 14 for the two spin-orientation). The author predicts this fits the maize seed nucleon model, where nucleons pack in a tetrahedral geometry. article amsmath, amssymb, booktabs

6 Energy Calibration and Shell-Gap Predictions from Unconverted Row Sums

6.1 Row Sums and Magic Numbers

Table 1: Row sums $S(r)$ evaluated with $x = 2$, and the two-spin-orientation magic numbers $M(r)$.

Row r	Unconverted row (evaluated)	$S(r)$	$M(r)$
1	$2^2 + 2$	6	2
2	$2^3 + 6^2 + 3$	47	6
3	$2^4 + 6^3 + 12^2 + 4$	380	14
4	$2^5 + 6^4 + 12^3 + 20^2 + 5$	3461	28
5	$2^6 + 6^5 + 12^4 + 20^3 + 30^2 + 6$	37482	50
6	$2^7 + 6^6 + 12^5 + 20^4 + 30^3 + 42^2 + 7$	484387	82
7	$2^8 + 6^7 + 12^6 + 20^5 + 30^4 + 42^3 + 56^2 + 8$	7353408	126
8	$2^9 + 6^8 + 12^7 + 20^6 + 30^5 + 42^4 + 56^3 + 72^2 + 9$	129104441	184
9	$2^{10} + 6^9 + 12^8 + 20^7 + 30^6 + 42^5 + 56^4 + 72^3 + 90^2 + 10$	2589967502	258
10	$2^{11} + 6^{10} + 12^9 + 20^8 + 30^7 + 42^6 + 56^5 + 72^4 + 90^3 + 110^2 + 11$	58757627063	350

6.2 Energy Ordering Within a Row

In the unconverted row, energy lessens from left to right, directly tied to the decrease in exponents. For the row $2^5 + 6^4 + 12^3 + 20^2 + 5$, the highest energy is carried by 2^5 (exponent 5), followed by 6^4 (exponent 4), and so on, down to the constant term 5 (exponent 0). Energy decreases horizontally across the row. This energy ordering explains why deformation can occur without net energy loss: after row division the row ends with a repeated pair (e.g. 5 + 5 or, after suitable replacement, 1 + 1), keeping the total energy intact.

6.3 Calibration of the Shell-Gap Formula

The difference between successive row sums is

$$\Delta S(r) = S(r) - S(r-1) \quad (r \geq 2).$$

The experimental single-particle shell gaps at the major magic numbers that coincide with $M(r)$ are

$$\begin{aligned} M = 28 : & \quad 6.0 \text{ MeV}, \\ M = 50 : & \quad 5.0 \text{ MeV}, \\ M = 126 : & \quad 4.0 \text{ MeV}. \end{aligned}$$

A power-law relation

$$\Delta E(r) = C [\Delta S(r)]^\alpha [M(r)]^\beta$$

is fitted to the three calibration points

$$(M = 28, \Delta S = 3081), \quad (M = 50, \Delta S = 34021), \quad (M = 126, \Delta S = 6869021).$$

Solving the resulting system of equations gives

$$C = 22.3 \text{ MeV}, \quad \alpha = 0.0456, \quad \beta = -0.503.$$

Thus the shell-gap energy determined solely from the row-sum difference and the magic number is

$$\boxed{\Delta E(r) = 22.3 [\Delta S(r)]^{0.0456} [M(r)]^{-0.503} \text{ MeV}.}$$

The formula reproduces the three calibration gaps exactly.

6.4 Predicted Shell Gaps

Table 2 lists the shell-gap energies computed for all available magic numbers. Each $\Delta S(r)$ is the difference of consecutive row sums from Table 1.

Table 2: Predicted shell-gap energies from the difference of consecutive row sums and the magic number.

$M(r)$	$\Delta S(r)$	ΔE_{model} (MeV)	Remarks
6	41	10.7	subshell
14	333	7.7	subshell
28	3 081	6.0	doubly magic ^{48}Ca region
50	34 021	5.0	doubly magic ^{100}Sn region
82	446 905	4.4	$1h_{11/2}$ gap (exp. 4.5–5.0 MeV)
126	6 869 021	4.0	doubly magic ^{208}Pb
184	121 751 033	3.8	predicted island of stability
258	2 460 863 061	3.7	superheavy island region
350	56 167 659 561	3.62	predicted

6.5 Worked Examples

Example 1: $M = 28$ (row 4)

$$\Delta S(4) = 3\,461 - 380 = 3\,081,$$

$$M(4) = 28.$$

$$\begin{aligned} [\Delta S]^{0.0456} &= \exp(0.0456 \times \ln 3\,081) \\ &= \exp(0.0456 \times 8.0326) \\ &= \exp(0.3663) = 1.442, \end{aligned}$$

$$\begin{aligned} [M]^{-0.503} &= \exp(-0.503 \times \ln 28) \\ &= \exp(-0.503 \times 3.3322) \\ &= \exp(-1.676) = 0.1870, \end{aligned}$$

$$\begin{aligned} \Delta E &= 22.3 \times 1.442 \times 0.1870 \\ &= 22.3 \times 0.2697 = 6.02 \approx 6.0 \text{ MeV}. \end{aligned}$$

Example 2: $M = 126$ (row 7)

$$\Delta S(7) = 7\,353\,408 - 484\,387 = 6\,869\,021,$$

$$M(7) = 126.$$

$$\begin{aligned} [\Delta S]^{0.0456} &= \exp(0.0456 \times \ln 6\,869\,021) \\ &= \exp(0.0456 \times 15.742) \\ &= \exp(0.7178) = 2.050, \end{aligned}$$

$$\begin{aligned} [M]^{-0.503} &= \exp(-0.503 \times \ln 126) \\ &= \exp(-0.503 \times 4.8363) \\ &= \exp(-2.433) = 0.0878, \end{aligned}$$

$$\begin{aligned} \Delta E &= 22.3 \times 2.050 \times 0.0878 \\ &= 22.3 \times 0.1800 = 4.01 \approx 4.0 \text{ MeV}. \end{aligned}$$

Example 3: $M = 350$ (row 10)

$$\begin{aligned}\Delta S(10) &= 58\,757\,627\,063 - 2\,589\,967\,502 = 56\,167\,659\,561, \\ M(10) &= 350.\end{aligned}$$

$$\begin{aligned}[\Delta S]^{0.0456} &= \exp(0.0456 \times \ln 56\,167\,659\,561) \\ &= \exp(0.0456 \times 24.7516) \\ &= \exp(1.1287) = 3.091,\end{aligned}$$

$$\begin{aligned}[M]^{-0.503} &= \exp(-0.503 \times \ln 350) \\ &= \exp(-0.503 \times 5.8579) \\ &= \exp(-2.9465) = 0.05255,\end{aligned}$$

$$\begin{aligned}\Delta E &= 22.3 \times 3.091 \times 0.05255 \\ &= 22.3 \times 0.1624 = 3.62 \text{ MeV}.\end{aligned}$$

7 Subshell Filling up to x^9

7.1 Important Note on the Yagah Proposed Nucleon Subshell Filling

This refers exclusively to nuclear shells governed by the strong nuclear force, distinct from atomic electron shells. The “s” and “p” labels follow a classification derived from the Yagah Prime Density Triangle statistics.

Island of Stability and Deformation via Repeated End Terms (n, n)

A central feature of the model is the interpretation of the repeated end integers ($n + n$) that appear at the close of each D row. These paired identical terms indicate paired nucleon groups that maintain structural stability. In the island of stability, nucleons do not lose energy when filling these levels. Instead, they become deformed while preserving the overall nucleon count. This deformation allows nucleons to be added to the next subshell without disrupting the closed-shell stability.

Explanation of Group Interpretation for Repeated End Pairs

For the central triangular terms the interpretation follows a straightforward multiplier \times base pattern. However, for the repeated end pairs (n, n), the grouping is more nuanced and does not mean simple multiplication.

Example (D5: 6 + 6): This is interpreted as $3n(6) \rightarrow 4s^6 = 12$. It is not 3×6 or 4×6 in the simple sense. Instead, it represents multiple groupings of the 4s orbital (here three groups), each associated with the repeated value 6, yielding the total of 12. The repeated ends reflect the paired/deformed nature in the island of stability.

Demonstration A: $x + 3 + 3$ (= 8 nucleons): Neutron: $n(2) \rightarrow 1s^2 = 2$, proton $p(3, 3) 2p^3 = 6$. Total: 8.

Demonstration B: $x + 6 + 12 + 5 + 5$ (= 30 nucleons): $x = n(2) \rightarrow 1s^2 = 2$; $6 = 2p(3) \rightarrow 2p^3 = 6$; $12 = 3n(4) \rightarrow 3s^4 = 12$; $5 + 5 = 2p(5) \rightarrow 2p^5 = 10$. Total 30. Use notation $A(s/p)B$, where A is the number of nucleon groups, B nucleons per group, $A \times B$ total nucleons. $x^0 = 1$ is the fundamental nucleon state.

\mathcal{D}_1 : $x^2 + 2 = 6$ $n(2) \rightarrow 1s^2 = 2$; $p(2) \rightarrow 2p^2 = 4$. Total 6.

\mathcal{D}_2 : $x + 3 + 3 = 8$ $n(2) \rightarrow 1s^2 = 2$; $p(3, 3) \rightarrow 2p^3 = 6$. Total 8.

\mathcal{D}_3 : $x + 6 + 4 + 4 = 16$ $x = n(2) \rightarrow 1s^2 = 2$; $6 = 2p(3) \rightarrow 2p^3 = 6$; $4 + 4 = 2n(4) \rightarrow 3s^4 = 8$. Total 16.

\mathcal{D}_4 : $x + 6 + 12 + 5 + 5 = 30$ $x = n(2) \rightarrow 1s^2 = 2$; $6 = 2p(3) \rightarrow 2p^3 = 6$; $12 = 3n(4) \rightarrow 3s^4 = 12$; $5 + 5 = 2p(5) \rightarrow 2p^5 = 10$. Total 30.

\mathcal{D}_5 : $x + 6 + 12 + 20 + 6 + 6 = 52$ $x = n(2) \rightarrow 1s^2 = 2$; $6 = 2p(3) \rightarrow 2p^3 = 6$; $12 = 3n(4) \rightarrow 3s^4 = 12$; $20 = 4p(5) \rightarrow 4p^5 = 20$; $6 + 6 = 3n(6) \rightarrow 4s^6 = 12$. Total 52.

\mathcal{D}_6 : $x + 6 + 12 + 20 + 30 + 7 + 7 = 84$ Fills to $4p^6 = 12$, $5s^6 = 12$, $3n(7) \rightarrow 5p^7 = 14$. Total 84.

\mathcal{D}_7 : $x + 6 + 12 + 20 + 30 + 42 + 8 + 8 = 128$ Fills to $5p^6 = 12$, $6s^6 = 12$, $4n(8) \rightarrow 6p^8 = 16$. Total 128.

\mathcal{D}_8 : $x + 6 + 12 + 20 + 30 + 42 + 56 + 9 + 9 = 186$ Fills to $6p^8 = 16$, $7s^6 = 12$, $5n(9) \rightarrow 7p^9 = 18$. Total 186.

\mathcal{D}_9 : $x + 6 + 12 + 20 + 30 + 42 + 56 + 72 + 10 + 10 = 260$ Fills to $7p^8 = 16$, $8s^6 = 12$, $6n(10) \rightarrow 8p^{10} = 20$. Total 260.

7.2 Complete Subshell Filling (Spin-Aware Version with p-Orbital Spins x,y,z)

From D2 onward, the spin(p orbital with x,y,z) version uses multiplication throughout from the beginning to the end.

1. **x2+ 2, x = 2**: First shell takes 6 nuclides. $n(2,2) + p(2) \rightarrow 2s^2, 2p^1$. Neutron: $2s^2 = 4$, Proton: $2p^1 = 2$. Total: 6.
2. **x + 3 + 3**: $n(2) + p(3,3) : 1s^2, 2p^3$
3. **x + 6 + 4 + 4**: $n(2) + 2p(3) + 2n(4) : n2 + 2n(4) + 2p(3) : 1s^2, 2s^4, 2p^3$
4. **x + 6 + 12 + 5 + 5**: $n2 + 3n(4) + 2p(3) + 2p(5) : 1s^2, 3s^4, 2p^3, 2p^5$
5. **x + 6 + 12 + 20 + 6 + 6**: $n2 + 3n(4) + 2n(6) + 2p(3) + 4p(5) : 1s^2, 3s^4, 2s^6, 2p^3, 4p_x^5$
6. **x+6+12+20+30+7+7**: $n2 + 3n(4) + 5n(6) + 2p(3) + 4p(5) + 2p(7) : 1s^2, 3s^4, 5s^6, 2p^3, 4p_x^5, 2p_y^7$
7. **x + 6 + 12 + 20 + 30 + 42 + 8 + 8**: $n2 + 3n(4) + 5n(6) + 2n(8) + 2p(3) + 4p(5) + 6p(7) : 1s^2, 3s^4, 5s^6, 2s^8, 2p^3, 4p_x^5, 6p_y^7$
8. **x + 6 + 12 + 20 + 30 + 42 + 56 + 9 + 9**: $n2+3n(4)+5n(6)+7n(8)+2p(3)+4p(5)+6p(7)+2p(9) : 1s^2, 3s^4, 5s^6, 7s^8, 2p^3, 4p_x^5, 6p_y^7, 2p_z^9$
9. **x + 6 + 12 + 20 + 30 + 42 + 56+72+10+10**.

Neutrons have odd base and even exponent; protons have the opposite. This mirrors isospin symmetry. Neutrons have odd base and even exponent; protons have the opposite. This mirrors isospin symmetry.

8 Deductions

1. The “s” shell corresponds to standing waves without spin; the “p” shell corresponds to standing waves with three orientations x, y, z . Completed shells have total angular momentum $J = 0$.
2. At the magic numbers the nucleus is in the island of stability.
3. In the island of stability the nucleons are deformed, with $2s, 2p$ adding shells while keeping nucleon numbers intact.
4. Neutrons have odd base and even exponent; protons have the opposite.
5. The author predicts this scenario best fits the maize seed nucleon model(or packing model as they stacked together).

9 Evidence Supporting Magic Numbers in Neutron Shells

1. San Jose State University applet-magic.com, Thayer Watkins. Incremental binding energy shows shell closures at 2, 8, 20, 28, 50, 82, 126. For two spin orientations: 2, 6, 14, 28, 50, 82, 126, 184.
2. Xavier Borges, “Magic numbers derived from a variable phase nuclear model,” *The General Science Journal*, 2005. Predicts magic number 184, which appears in the Yagah sequence.
3. Wikipedia, “Magic number (physics)”. Double magic nuclei include ^4He , ^{16}O , ^{48}Ca , ^{56}Ni , ^{132}Sn , ^{208}Pb . Island of stability predicted around $Z = 114\text{--}126$, $N = 184$.

4. O. Haxel, J. H. D. Jensen, and H. E. Suess, “On the ‘Magic Numbers’ in Nuclear Structure,” *Phys. Rev.* **75**, 1766 (1949).
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10 Conclusion

The Yagah prime density triangle provides a combinatorial construction whose row division yields the magic numbers for two spin orientations, and after replacing repeated end integers, the standard nuclear magic numbers 2, 8, 20, 28, 50, 82, 126, 184, . . . The generating function for row sums and the recurrence $M = m \pm [(n_1 n_2) + 2]$ give explicit computational tools. The subshell filling and deductions align with a tetrahedral packing model for nucleons.

Data Availability Statement

The data and code that support the findings of this study are openly available in Zenodo at <https://doi.org/10.5281/zenodo.20233053> and in Mathscidoc at [mathscidoc:2605.13001](https://mathscidoc.org/mathscidoc:2605.13001).

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