

Logarithmic Root-Sum-Product Extractor or Half-Life Quadratic Formula

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Abstract

This paper introduces a new method of finding the sum and product of roots of quadratic equations. The approach adapts the half-life concept from radioactive decay in Physics and Chemistry. During the derivation of the quadratic formula by completing the square, the step where we add half the coefficient of x to both sides is replaced by the half-life relation $1/2 = \ln 2/k$, leading to $k = 2 \ln 2$. This substitution produces an auxiliary formula that yields two values x_1 and x_2 whose sum equals the Vieta sum. A second relation involving $\ln 2$ then recovers the exact Vieta product. The method, called the Logarithmic Root-Sum-Product Extractor (or Half-Life Quadratic Formula), gives results identical to the conventional method while creating a strong link between mathematics and science.

KEYWORDS: Quadratic equation, half-life, $\ln 2$, sum of roots, product of roots, completing the square, radioactive decay.

1 Introduction

This paper is based on the use of a new method to find the sum and product of roots of a quadratic equation. If we use the conventional quadratic formula, we arrive at the same sum and product of roots that we may obtain from this new approach or any other known method.

This new approach makes use of the half-life concept by substituting into the conventional quadratic equation $ax^2 + bx + c = 0$ during the completing-the-square process. The method provides an alternative pedagogical pathway that connects algebraic manipulation with exponential decay principles, thereby demonstrating the interconnectedness of mathematical and scientific disciplines. By introducing logarithmic constants into the derivation of the quadratic formula, students and researchers alike can appreciate how concepts from different fields can inform and enrich each other.

2 The Conventional Quadratic Equation

Consider the general quadratic equation

$$ax^2 + bx + c = 0 \quad (a \neq 0). \quad (1)$$

If α and β are the roots, then by Vieta's formulas:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}. \quad (2)$$

3 Construction of the Half-Life Quadratic Formula

We start with the standard derivation of the quadratic formula by completing the square, but substitute the half-life relation at the key step.

Step 1 – Standard completing the square

$$ax^2 + bx + c = 0 \Rightarrow x^2 + \frac{b}{a}x = -\frac{c}{a}. \quad (3)$$

Add $\left(\frac{b}{2a}\right)^2$ to both sides:

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}. \quad (4)$$

The left side is a perfect square:

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}. \quad (5)$$

Step 2 – Introducing the half-life analogy

In radioactive decay, the half-life relation is

$$\frac{N}{N_0} = \frac{1}{2} = e^{-kt_{1/2}} \Rightarrow t_{1/2} = \frac{\ln 2}{k}. \quad (6)$$

Set $\frac{1}{2} = \frac{\ln 2}{k} \Rightarrow k = 2 \ln 2$. Let $L = \ln 2$. Then $k = 2L$.

Multiply the equation by $2L$ before completing the square:

$$2Lx^2 + \frac{2Lb}{a}x = -\frac{2Lc}{a}. \quad (7)$$

Add $\left(\frac{Lb}{a}\right)^2$ to both sides and simplify to get the auxiliary formula:

$$x = \frac{-bL \pm \sqrt{2L(b^2 - 2ac)}}{2aL}. \quad (1a)$$

In terms of $\ln 2$ directly:

$$x = \frac{-b \ln 2 \pm \sqrt{2 \ln 2 (b^2 - 2ac)}}{2a \ln 2}. \quad (1b)$$

Let $s = x_1 + x_2$ and $p = x_1 x_2$. The actual product is recovered from:

$$\frac{s^2}{2} + L \left(p - \frac{s^2}{4}\right) = \frac{c}{a}. \quad (2)$$

4 Proof of the Sum of Roots

Adding the two solutions from (1a) cancels the square-root terms:

$$s = x_1 + x_2 = \frac{-bL + \sqrt{\dots}}{2aL} + \frac{-bL - \sqrt{\dots}}{2aL} = -\frac{b}{a}, \quad (8)$$

which is identical to Vieta's sum $\alpha + \beta$.

5 Proof of the Product of Roots

Substituting $s = -b/a$ and $p = x_1x_2$ from (1a) into equation (2) and simplifying yields exactly c/a , matching Vieta's product $\alpha\beta$.

6 Worked Examples

We use $L = \ln 2 \approx 0.693147$.

Example 1: $x^2 - 5x + 6 = 0$ ($a = 1$, $b = -5$, $c = 6$). Expected: sum = 5, product = 6.

Formula (1a):

$$x = \frac{-(-5)L \pm \sqrt{2L((-5)^2 - 2 \cdot 1 \cdot 6)}}{2 \cdot 1 \cdot L} = \frac{5L \pm \sqrt{2L(25 - 12)}}{2L}. \quad (9)$$

This gives $x_1 \approx 5.5623$, $x_2 \approx -0.5623$. $s = 5.0000$, $p \approx -3.1275$. Product recovered: $\frac{s^2}{2} + L\left(p - \frac{s^2}{4}\right) = 6.0000$.

Example 2: $2x^2 + 3x - 2 = 0$ ($a = 2$, $b = 3$, $c = -2$). Expected: sum = -1.5, product = -1.

Formula (1a):

$$x = \frac{-3L \pm \sqrt{2L(3^2 - 2 \cdot 2 \cdot (-2))}}{2 \cdot 2 \cdot L} = \frac{-3L \pm \sqrt{2L(9 + 8)}}{4L}. \quad (10)$$

This gives $x_1 \approx 1.00093$, $x_2 \approx -2.50093$. $s = -1.50000$, product recovered = -1.0000.

Example 3: $x^2 - 7x + 10 = 0$ ($a = 1$, $b = -7$, $c = 10$). Expected: sum = 7, product = 10.

Formula (1a):

$$x = \frac{7L \pm \sqrt{2L(49 - 20)}}{2L} = \frac{7L \pm \sqrt{2L \cdot 29}}{2L}. \quad (11)$$

This gives $x_1 \approx 8.0740$, $x_2 \approx -1.0740$. $s = 7.0000$, product recovered = 10.0000.

Example 4: $3x^2 - 10x + 3 = 0$ ($a = 3$, $b = -10$, $c = 3$). Expected: sum = $10/3 \approx 3.33333$, product = 1.

Formula (1a):

$$x = \frac{10L \pm \sqrt{2L(100 - 18)}}{6L} = \frac{10L \pm \sqrt{2L \cdot 82}}{6L}. \quad (12)$$

This gives $x_1 \approx 4.2300$, $x_2 \approx -0.8967$. $s = 3.3333$, product recovered = 1.0000.

Example 5: $x^2 + 4x + 3 = 0$ ($a = 1$, $b = 4$, $c = 3$). Expected: sum = -4, product = 3.

Formula (1a):

$$x = \frac{-4L \pm \sqrt{2L(16 - 6)}}{2L} = \frac{-4L \pm \sqrt{2L \cdot 10}}{2L}. \quad (13)$$

This gives $x_1 \approx 0.6858$, $x_2 \approx -4.6858$. $s = -4.0000$, product recovered = 3.0000.

7 Conclusion

The Logarithmic Root-Sum-Product Extractor, also known as the Half-Life Quadratic Formula, provides a fresh way to find the sum and product of roots of quadratic equations. By substituting the half-life relation $1/2 = \ln 2/k$ at the completing-the-square stage, we obtain auxiliary values whose sum and derived product relation exactly reproduce Vieta's formulas. This approach strengthens the link between mathematics and concepts from radioactivity in Physics and Chemistry and can serve as an engaging teaching tool.

The method demonstrates that mathematical formulas need not be derived in isolation but can emerge from interdisciplinary connections. The introduction of logarithmic constants into the quadratic formula derivation offers an alternative perspective that may help students understand both the algebraic structure of quadratic equations and the exponential processes found

in nature. The substitution principle introduced here has since been extended beyond the algebraic setting: in a companion work, the same half-life substitution $1/2 \rightarrow \ln 2/k$ is carried into quantum gauge theory, where it produces a kinetic–half-life correspondence and an analytical derivation of the Yang–Mills mass gap (Yagah, 2026). That result—a mass gap of 1.75 GeV and a flux-tube thickness of 0.352 GeV^{-1} , consistent with the lattice value 0.343 GeV^{-1} —demonstrates that the substitution principle is not merely a pedagogical device but a transferable mathematical structure with genuine reach into theoretical physics, thereby reinforcing the interdisciplinary value of the present method.

References

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