THE OWARI SERIES: AN ILLUSTRATIVE EXPLORATION OF SUMMATION AND PARTIAL FRACTIONS IN A 2-OR-3 SCORING VARIANT

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Abstract

This paper presents a recreational mathematical exploration derived from the traditional African board game Owari (Mancala). The study examines a specific variant where a score is achieved only if a player casts marbles totaling exactly two or three into the opponent's home, with no scoring occurring in one's own home. Both players sow anticlockwise relative to their own perspective; because they sit opposite one another, their physical trajectories on the board are opposite. It is conjectured that this opposing dynamic may produce alternating signs in a telescoping series, though this remains an open question (see Section 5.6). The game's scoring sequence is illustrated using arithmetic progressions (AP), geometric progressions (GP), and telescoping series as analogies. The paper analyzes two distinct illustrative games: the first is a concrete 10-round example whose residual happens to be 1/12; the second is an illustrative game with final scores of 23 and 25, where the cumulative scores sum to 48. By representing score distributions through partial fractions, the study illustrates possible analogies with telescoping series. Additional variants including 1-or-4 scoring and symmetric sowing are introduced as worked examples. The paper does not claim to derive the sequences from the rules; rather, it illustrates possible mathematical analogies.

KEYWORDS: Owari (Mancala), Recreational Mathematics, Partial Fractions, Telescoping Series, Geometric Progression, Arithmetic Progression, Finite Game Theory.

1 INTRODUCTION

The game of Owari, a variant of Mancala, involves strategic seed-sowing across twelve holes, with a total of forty-eight marbles [1, 4, 7]. In standard play, players cast from their respective sides, aiming to capture marbles from their opponent. However, the specific variant explored in this paper introduces a recreational mathematical constraint: points are exclusively scored when a cast results in a total of exactly two or three marbles in an opponent's hole. Captures of four or above, as well as any counts in one's own home, are nullified. This paper does not attempt a formal derivation of the game's sequences from the rules. Instead, it illustrates possible analogies between the scoring patterns and standard mathematical series [2, 5].

Rules of the Game and Board Evolution

The initial board configuration consists of 4 marbles in each of the 12 holes (48 total). On a player's turn, they select a hole on their side, remove all marbles from it, and sow them one by one into subsequent holes in their respective direction. Both players sow anticlockwise relative to their own perspective. Because Player A and Player B sit opposite one another, their sowing

trajectories move in physically opposite directions around the board. A capture occurs only if the final marble placed lands in an opponent's hole, making the total marble count in that hole exactly 2 or 3. Captured marbles are permanently removed from the board and added to the player's score. If the count is anything else (1, 4, or more), no score is awarded and the marbles remain on the board.

The initial board configuration is shown below:

```
        Player A (Anticlockwise)
+---+---+---+---+---+---+
| 4 | 4 | 4 | 4 | 4 | 4 |
+---+---+---+---+---+---+
+---+---+---+---+---+---+
| 4 | 4 | 4 | 4 | 4 | 4 |
+---+---+---+---+---+---+
        Player B (Anticlockwise)
```

2 FORMAL DEFINITION OF THE 2-OR-3 VARIANT

Definition 2.1 — Board State. Let $A_k = [a_1, a_2, a_3, a_4, a_5, a_6]_k$ be Player A's board at round k and $B_k = [b_1, b_2, b_3, b_4, b_5, b_6]_k$ be Player B's board at round k. Let S_k^A and S_k^B be cumulative scores. The conservation law is:

$$\sum_{i=1}^{6} a_i + \sum_{i=1}^{6} b_i + S_k^A + S_k^B = 48$$

Definition 2.2 — Sowing Direction. Both players sow anticlockwise from their own perspective. Because they are seated opposite one another, Player A's anticlockwise motion and Player B's anticlockwise motion are physically opposite in the global board layout. It is conjectured that this opposing dynamic may produce alternating signs in a telescoping expansion, though this is not demonstrated in this paper (see Section 5.6).

Definition 2.3 — Capture Function (2-or-3 Scoring). Let m be the final marble count in the target hole h. The capture function is:

$$C(m) = \begin{cases} m & \text{if } m \in \{2, 3\} \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.4 — Telescoping Analogy. A capture of 2 is associated with the telescoping term:

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}.$$

For a capture of 2, $n = 1$, which gives $1/(1 \times 2) = 1 - 1/2$. For a capture of 3, $n = 2$, which gives $1/(2 \times 3) = 1/2 - 1/3$. This is an illustrative analogy only; no derivation from the game's rules is claimed.

Definition 2.5 — Terminal Condition. The game terminates when $\sum_{i=1}^{6} a_i + \sum_{i=1}^{6} b_i \leq 4$. The residual R is defined as $R = 48 - (S_N^A + S_N^B)$, where N is the terminal round.

Justification of Finite Termination: Because captured marbles are permanently removed from the board upon scoring, the total number of marbles on the board is non-increasing and strictly decreases on capture rounds. In the observed games, this led to termination.

3 GAME 1: BOARD CONSTRUCTION AND ILLUSTRATIVE EXAMPLE

Let the board be represented as a 12-tuple, where Player A plays from one side and Player B from the other. The initial setup consists of 4 marbles in all 12 holes, totaling 48 marbles. The game proceeds sequentially. The board states in the table below are illustrative hypothetical play; they are not accompanied by a move-by-move verification.

Table 1: Game 1 Progression (Illustrative)

Round	A (Anticlockwise)	B (Anticlockwise)	A Score (per round)	B Score (per round)	Cumulative A	Cumulative B	Board + Score Check
1	[5, 5, 5, 4, 0, 5]	[5, 5, 5, 4, 0, 5]	0	0	0	0	$24 + 24 + 0 + 0 = 48$
2	[6, 6, 1, 6, 0, 6]	[6, 6, 5, 4, 0, 0]	0	2	0	2	$25 + 21 + 0 + 2 = 48$
3	[7, 7, 0, 7, 0, 6]	[6, 6, 5, 0, 1, 1]	0	0	0	2	$27 + 19 + 0 + 2 = 48$
4	[7, 7, 0, 7, 0, 0]	[7, 7, 6, 0, 1, 0]	4	0	4	2	$21 + 21 + 4 + 2 = 48$
5	[8, 8, 1, 1, 1, 1]	[8, 8, 0, 2, 1, 1]	2	0	6	2	$20 + 20 + 6 + 2 = 48$
6	[9, 1, 0, 0, 0, 2]	[9, 0, 2, 1, 2, 2]	3	9	9	11	$12 + 16 + 9 + 11 = 48$
7	[1, 0, 1, 1, 1, 3]	[10, 1, 0, 0, 2, 0]	5	3	14	14	$7 + 13 + 14 + 14 = 48$
8	[1, 0, 0, 0, 0, 3]	[0, 2, 1, 1, 3, 1]	0	8	14	22	$4 + 8 + 14 + 22 = 48$
9	[2, 1, 0, 0, 0, 0]	[1, 0, 0, 1, 0, 2]	5	0	19	22	$3 + 4 + 19 + 22 = 48$
10	[1, 0, 1, 0, 0, 0]	[1, 0, 0, 1, 0, 0]	0	3	19	25	$2 + 2 + 19 + 25 = 48$
Final	**[0, 0, 0, 0, 0, 0]**	**[1, 1, 0, 1, 1, 0]**	**0**	**0**	**19**	**25**	$\mathbf{0 + 4 + 19 + 25 = 48}$

Note on conservation law: The final column demonstrates that the conservation law holds across all rounds when using cumulative scores.

The per-round scores for Player A are:

$$a_1 = 0,\ a_2 = 0,\ a_3 = 0,\ a_4 = 4,\ a_5 = 2,\ a_6 = 3,\ a_7 = 5,\ a_8 = 0,\ a_9 = 5,\ a_{10} = 0.$$

The per-round scores for Player B are:

$$b_1 = 0,\ b_2 = 2,\ b_3 = 0,\ b_4 = 0,\ b_5 = 0,\ b_6 = 9,\ b_7 = 3,\ b_8 = 8,\ b_9 = 0,\ b_{10} = 3.$$

The cumulative scores are:

$$T_A = \sum_{n=1}^{10} a_n = 0 + 0 + 0 + 4 + 2 + 3 + 5 + 0 + 5 + 0 = 19,$$

$$T_B = \sum_{n=1}^{10} b_n = 0 + 2 + 0 + 0 + 0 + 9 + 3 + 8 + 0 + 3 = 25.$$

The total marbles in the system is 48, leaving 4 residual marbles on Player B's board at the terminal state. The game is a concrete 10-round example.

3.1 Summation for Game 1

For this concrete 10-round example, the upper bound is $N = 10$:

$$T_A = \sum_{n=1}^{10} a_n = 19, \qquad T_B = \sum_{n=1}^{10} b_n = 25.$$

Given the terminal state where 4 marbles remain on the board (a residual of $4/48 = 1/12$), the captured fraction is $44/48 = 11/12$. This illustrates that the terminal state is consistent with the residual marbles. It is not claimed as a general proof of finite termination.

3.2 Worked Examples of 2-or-3 Scoring as Telescoping Analogies

Capture of 2: Hole contains 1 marble, player sows 1 marble, total becomes 2. Captured. Telescoping analogy: $\frac{1}{1\times 2} = \frac{1}{1} - \frac{1}{2}$.

Capture of 3: Hole contains 2 marbles, player sows 1 marble, total becomes 3. Captured. Telescoping analogy: $\frac{1}{2\times 3} = \frac{1}{2} - \frac{1}{3}$.

Failed capture of 4: Hole contains 3 marbles, player sows 1, total becomes 4. No capture. Score 0.

3.3 Conservation Law

For every round k:

$$A_{\text{board}}(k) + B_{\text{board}}(k) + A_{\text{score}}(k) + B_{\text{score}}(k) = 48.$$

When $A_{\text{board}} + B_{\text{board}} = 4$, the game stops, leaving residual 1/12.

4 ILLUSTRATIVE GAME 2: PROBLEMS AND CLOSURE

4.1 Problem Formulation for Game 2

Table 2 illustrates an Owari game played by two students, A and B. The problem requires (a) predicting the index at which the partial fraction statement is true, and (b) observing the rounds at which the game mathematically reached closure.

Table 2: Game 2 Scoring Progression (Illustrative)

Round	**1**	**2**	**3**	**4**	**5**	**Final Score**
A Scores	5	2	2	8	6	**23**
B Scores	5	3	6	8	3	**25**

4.2 Predicting the Index of the Statement

The game's cumulative scoring is modeled using the term $\frac{23}{7n+6}$. To find the index at which this expression equals Player A's score fraction $A = \frac{23}{48}$, the denominator is equated to the total number of marbles in the system:

$$7n + 6 = 48 \implies 7n = 42 \implies n = 6$$

At $n = 6$, the term yields exactly $\frac{23}{48}$. Similarly, for Player B, the expression $\frac{25}{7n+6}$ yields $B = \frac{25}{48}$ at $n = 6$. Therefore, the statement is mathematically valid at the index $n = 6$.

4.3 Observation of Sequence Closure

Rounds 26 (for Player A) and 29 (for Player B) are observed terminal rounds from the played game. The marble sequences align with these observed rounds: for A, $1104 = 23 \times 48$, and for B, the sequence aligns with a denominator of 42. These are observed results from the sequence termination, not derived from a recurrence relation.

4.4 Physical Interpretation (Conjecture)

Player B ended at round 29, while Player A ended at round 26. It is conjectured that the 3-round difference corresponds to the 3 occasions where B transferred one marble to A, allowing A to restart the sequence when short of marbles. This is offered as speculation, not proof. The total conservation law holds at closure for Game 2:

$$T_A + T_B = 23 + 25 = 48$$

5 FURTHER SERIES AND VARIANTS

5.1 Geometric Series Problem

Find the round of the Owari series at which the term equals 1/8, where the terms are given by $1/2^n$ starting from r_0. Hence find the sum up to the 5th round.

Solution. The terms of the geometric sequence starting from r_0 are:

$$r_0 = 1, \quad r_1 = \frac{1}{2}, \quad r_2 = \frac{1}{4}, \quad r_3 = \frac{1}{8}.$$

Hence the round is r_3. The sum to 5 terms is:

$$S_5 = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}.$$

Common ratio $r = 1/2$, so it is a GP with first term $a = 1$:

$$S_5 = \frac{1\left(1 - \left(\frac{1}{2}\right)^5\right)}{1 - \frac{1}{2}} = \frac{1 - \frac{1}{32}}{\frac{1}{2}} = 2\left(\frac{31}{32}\right) = \frac{62}{32} = \frac{31}{16}.$$

5.2 General Series Problem

Find the round of the Owari series at which the term equals 7/16, where the terms are given by $n/2^{n-3}$ starting from r_0. Hence find the sum up to that round.

Solution. Using the formula $n/2^{n-3}$ with $n = 0, 1, 2, \ldots$, the terms are:

$$r_0 = 0, \quad r_1 = 4, \quad r_2 = 4, \quad r_3 = 3, \quad r_4 = 2, \quad r_5 = \frac{5}{4}, \quad r_6 = \frac{3}{4}, \quad r_7 = \frac{7}{16}.$$

Hence the round is r_7. The sequence is neither an AP nor a GP. The direct sum of the 7 non-zero terms is:

$$S = 4 + 4 + 3 + 2 + \frac{5}{4} + \frac{3}{4} + \frac{7}{16}.$$

Converting all terms to a common denominator of 16:

$$4 = \frac{64}{16}, \quad 4 = \frac{64}{16}, \quad 3 = \frac{48}{16}, \quad 2 = \frac{32}{16},$$
$$\frac{5}{4} = \frac{20}{16}, \quad \frac{3}{4} = \frac{12}{16}, \quad \frac{7}{16} = \frac{7}{16}.$$

Sum of numerators: $64 + 64 + 48 + 32 + 20 + 12 + 7 = 247$. Hence $S = 247/16 = 15.4375$.

5.3 AP Example on Owari Sowing

If a player sows consecutively 4, 5, 6, 7, 8 marbles across holes, this forms an arithmetic progression with first term $a = 4$ and common difference $d = 1$. The sum of the first 5 terms is:

$$S_5 = \frac{5}{2}\big(2 \times 4 + (5 - 1) \times 1\big) = \frac{5}{2}(12) = 30.$$

5.4 GP Example from Residual Halving

If the residual marbles halve on each final sweep (24, 12, 6, 3), this forms a GP with $a = 24$ and $r = 1/2$ [6]. The sum of the first 4 terms is $S_4 = 24 + 12 + 6 + 3 = 45$. The sum to infinity is:

$$S_\infty = \frac{a}{1 - r} = \frac{24}{1 - \frac{1}{2}} = 48.$$

This illustrates closure, approaching the total marble count of 48.

5.5 Variant: 1-or-4 Scoring

In this variant, capture occurs only when the total is 1 or 4. The capture function is:

$$C(m) = \begin{cases} m & \text{if } m \in \{1, 4\} \\ 0 & \text{otherwise} \end{cases}$$

Partial fraction model. Consider $\frac{1}{n(n+3)}$. Decomposing:

$$\frac{1}{n(n+3)} = \frac{1}{3}\left(\frac{1}{n} - \frac{1}{n+3}\right).$$

The telescoping sum is:

$$\sum_{n=1}^{N} \frac{1}{n(n+3)} = \frac{1}{3}\left(1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{N+1} - \frac{1}{N+2} - \frac{1}{N+3}\right).$$

As $N \to \infty$, the sum converges to $\frac{1}{3}\left(\frac{11}{6}\right) = \frac{11}{18}$. This is an isolated identity, not a derivation from the game.

5.6 Remark: Symmetric Sowing

If both players sow anticlockwise (relative to themselves), the sum becomes:

$$\sum \frac{23 + 25}{n(n+12)}$$

or, more generally, for a full board:

$$\sum \frac{48}{n(n+12)}$$

Because they sit opposite one another, their trajectories are physically opposite. It is conjectured that this opposing dynamic may yield faster cancellation due to alternating signs in the telescoping expansion. This is a pure conjecture, not demonstrated in this paper. A formal proof of this behavior is left for future work.

6 CONCLUSION

The mathematical analysis of the 2-or-3 scoring variant of Owari illustrates possible analogies between recreational gameplay and formal mathematical series. In Game 1, the 10-round example has per-round scores summing to $T_A = 19$ and $T_B = 25$, with residual marbles consistent with the fraction 1/12. In Illustrative Game 2, the scores of 23 and 25 show that the statement $A = 23/48$ and $B = 25/48$ holds at $n = 6$, and the game reaches closure with Player A finishing at round 26 and Player B at round 29. The difference in rounds is conjectured to correspond to the 3 occasions where B had to give out one marble to allow A to continue. This is speculation, not a derived result. Additional variants — 1-or-4 scoring and symmetric sowing — are included as worked examples. This paper does not claim to derive the sequences from the rules; it illustrates possible mathematical analogies between the scoring patterns and standard series.

7 LIMITATIONS

The series presented in this paper are analogies, not derivations from the rules of the game. The telescoping mappings are illustrative rather than mathematically forced. The board states

in Table 1 are illustrative hypothetical play and are not accompanied by a move-by-move verification. Game 2's closure at rounds 26 and 29 is an observed result from play, not a derived consequence of a recurrence relation. The claim in Section 5.6 about alternating signs is a conjecture, not a proven result. The paper does not exhibit a recurrence, generating function, or closed-form expression that forces termination at particular indices. The references are primarily popular and historical works, not formal treatments of the specific mathematical modelling attempted here. This note is offered as a recreational mathematical exploration rather than a formal contribution.

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